The Košice meteoroid investigation: from trajectory data to analytic model

Daria Kuznetsova[1,2], **Maria Gritsevich**[3,1,2,4], **Vladimir Vinnikov**[1]

[1]**Dorodnicyn Computing Centre RAS, Moscow, Russia**
morven9@yandex.ru
maria.gritsevich@fgi.fi
vvinnikov@list.ru

[2]**Moscow State University of Geodesy and Cartography (MIIGAiK), Moscow, Russia**

[3]**Finnish Geodetic Institute, Masala, Finland**

[4]**Ural Federal University, Ekaterinburg, Russia**

Impact rate estimates for the upper atmosphere are significantly higher than for the Earth's surface due to the presence of the atmosphere. Thus to account for this properly, one needs to model drag and ablation processes along the atmospheric trajectory (e.g. Bland and Artemieva, 2003). The best way to validate the resulting model is to apply it to meteorite-producing fireballs with a complete observational record. We consider the recent meteorite fall – Košice (2010). In this investigation, we propose a special model based on the analytical solution of the drag and mass-loss equations (Gritsevich, 2009; Gritsevich et al., 2012). Using the available trajectory data (Borovička et al., 2013), two key dimensionless parameters (the ballistic coefficient and mass loss parameter) are obtained which allow us to describe the mass and velocity changes of the main fragment of the meteoroid entering the atmosphere, as well as to estimate the pre-atmospheric meteoroid mass. Good agreement between the calculated functions and real trajectory characteristics is shown. We also apply statistical methods to describe the fragmentation process and provide insights into the pre-atmospheric meteoroid shape (Vinnikov et al., 2014). Furthermore, the most probable scenario suggests that the Košice meteoroid, prior to further extensive fragmentation in the lower atmosphere, consisted of two independent pieces with cumulative residual masses of approximately 2 kg and 9 kg respectively (Gritsevich et al., 2014a). The conducted analysis leads to the conclusion that two to three larger Košice fragments of 500-1000 g each should exist in addition to the already reported meteorite finds.

1 Introduction

This study is focused on the development of a theoretical model which can describe the trajectory of a meteoroid after the entry in the atmosphere and predict the consequences of the impact. The existing data on meteoroid entry in the atmosphere should be used to show its validity once the model is constructed. In order to show the application of our model to real data, a recent fireball event was chosen.

The Košice meteorite fall is the result of the fireball event over central-eastern Slovakia which occurred on February 28, 2010. The landing area was successfully computed using the data from the surveillance cameras operating in Hungary (Borovička et al., 2013), and a meteorite recovery became possible[1]. 218 fragments of the Košice meteorite, with a total mass of 11.285 kg, have been documented (Gritsevich et al., 2014a). Laboratory analysis showed that the Košice meteorite (*Figure 1*) is an ordinary H5 chondrite with average bulk and grain densities of 3.43 and 3.79 g/cm^3 respectively (Kohout et al., 2014).

2 Mathematical model

The main equations for the motion of a body entering the terrestrial atmosphere are:

$$M\frac{dV}{dt} = -\frac{1}{2}c_d\rho_a V^2 S$$

$$\frac{dh}{dt} = -V\sin\gamma$$

$$H^*\frac{dM}{dt} = -\frac{1}{2}c_h\rho_a V^3 S$$

where M - body mass, V - body velocity, t - time, h - height above the planetary surface, H^* - effective destruction enthalpy, S – cross-section area of the body, ρ_a - atmospheric density, c_d - drag coefficient, and c_h – coefficient of heat exchange.

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[1] http://www.imo.net/imc2010/talks/Kaniansky.pdf Tóth Juraj, Svoreň Ján, Borovička Jiří, Spurný Pavel, Igaz Antal, Porubčan V., Kornoš L., Husárik Marek, Krišandová Z., Vereš P., Kaniansky S. (2010). "Meteorite Košice – The Fall in Slovakia", International Meteor Conference, IMC 2010, Sep. 16-19, 2010, Armagh, UK.

Figure 1 – Four fragments of Košice meteorite. Photo credit: T. Kohout, University of Helsinki, Finland.

We introduce the dimensionless quantities as follows:

$$M = M_e\, m, V = V_e\, v, h = h_0\, y, \rho_a = \rho_0\, \rho, S = S_e\, s \ (1)$$

where M_e is the pre-entry mass, V_e – pre-entry velocity, h_0 – height of the homogeneous atmosphere, ρ_0 – atmospheric density near the planetary surface.

Then from the initial equations one can obtain the equations for dimensionless mass and height as functions of velocity:

$$m\frac{dv}{dy} = \frac{1}{2}c_d\,\frac{\rho_0 h_0 S_e}{M_e}\,\frac{\rho v s}{\sin\gamma}$$

$$\frac{dm}{dy} = \frac{1}{2}c_h\,\frac{\rho_0 h_0 S_e}{M_e}\,\frac{V_e^2 \rho v^2 s}{H^* \sin\gamma}$$

We consider the isothermal atmosphere:

$$\rho = \exp(-y)$$

and we use the following relation between the cross-section area and the body mass:

$$s = m^{\mu}$$

where μ=const – the parameter characterizing the possible rotation role during the flight.

Then mass and height above the surface for a meteoroid entering the terrestrial atmosphere can be represented as the following functions of velocity:

$$m = \exp\left(-\frac{\beta}{1-\mu}\left(1-v^2\right)\right) \qquad (2)$$

$$y = \ln\alpha + \beta - \ln\frac{\Delta}{2} \qquad (3)$$

where

$$\Delta = Ei(\beta) - Ei(\beta v^2),\ Ei(x) = \int_{-\infty}^{x} \frac{e^z dz}{z}$$

and two dimensionless parameters are introduced:

the ballistic coefficient

$$\alpha = \frac{1}{2}c_d\,\frac{\rho_0 h_0 S_e}{M_e \sin\gamma} = \frac{1}{2}c_d\,\frac{\rho_0 h_0 A_e}{M_e^{1/3}\rho_b^{2/3}\sin\gamma}$$

where A_e – pre-atmospheric shape factor and ρ_b – meteoroid density, and the mass loss parameter

$$\beta = \frac{1}{2}(1-\mu)\frac{c_h V_e^2}{c_d H^*} .$$

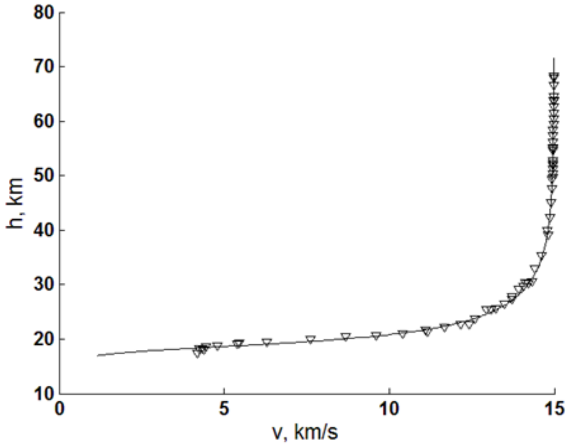

Figure 2 – Height vs velocity (in dimensional form) for the Košice meteorite case. Symbols – observational data (Borovička et al, 2013) (for the main fragment), full line – calculated trajectory according to (3) and (1).

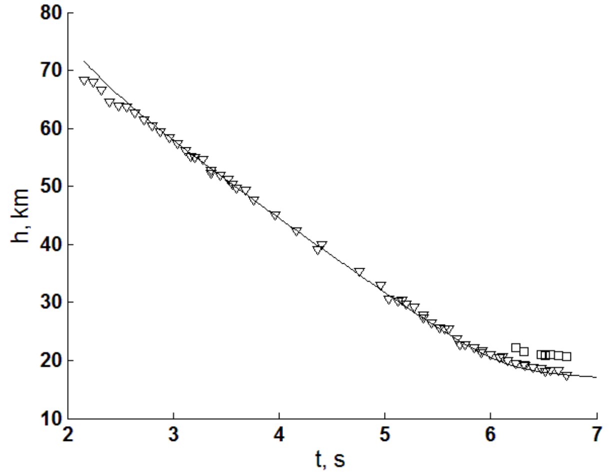

Figure 3 – Height change with time (in dimensional form) for the Košice case. Symbols – observational data (Borovička et al, 2013) (triangles – for the main fragment, squares – for the second fragment), full line – calculated trajectory.

For every meteor event one can find corresponding α and β values from the observational data (observed at certain points along the trajectory with corresponding heights h_i and velocities V_i, $i = 1, 2, ..., n$) according to the method described in (Gritsevich, 2007). Application of this method to the case of Košice is explained in (Gritsevich et al., 2014b). Thus, after obtaining the values of α and β from the observational data, it is possible to calculate the trajectory and follow the change in meteoroid mass based on (2) and (3). In *Figure 2* the obtained height-velocity relation is shown and in *Figure 3* the corresponding height-time dependence is shown, where symbols denote the observational data and the line is the calculated trajectory. A good agreement can be seen between observational and calculated data, as well as in the cases of Přibram, Lost City, Innisfree, and Neuschwanstein falls (Gritsevich, 2008).

3 Mass estimation and fragment distribution

The value of the ballistic coefficient α found using the observational data can be used to estimate the dynamic mass of the main fragment of the Košice meteoroid:

$$M_e = \left(\frac{1}{2} c_d \frac{\rho_0 h_0}{\alpha \sin\gamma} \frac{A_e}{\rho_m{}^{2/3}}\right)^3$$

According to the fragment mass data (Gritsevich et al., 2014a), we can also use statistical methods for additional analysis to describe the Košice fragmentation process. In particular, it was found that bimodal Weibull, bimodal Grady and bimodal lognormal distributions are the most appropriate (Gritsevich et al., 2014b). As an example of fragment distribution the approximation by the bimodal Weibull distribution is shown in Figure 4:

$$F_W(m,\omega,\gamma_1,\mu_1,\gamma_2,\mu_2) =$$

$$= \omega\left[1-\exp\left(-(m/\mu_1)^{\gamma_1}\right)\right]+$$

$$+(1-\omega)\left[1-\exp\left(-(m/\mu_2)^{\gamma_2}\right)\right]$$

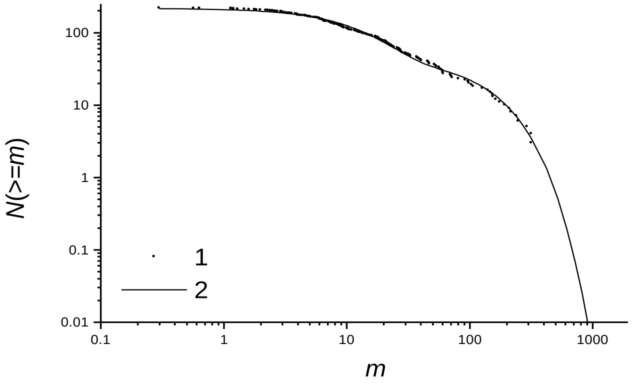

Figure 4 – Complementary cumulative number of fragments $N(\geq m)$ vs m (decimal logarithm scale) for the sample. 1 – Observed data, 2 – Bimodal Weibull distribution with the weighting factor ω=0.8, $\gamma_1=\gamma_2$=1.14 and μ_1=13.1, μ_2=140.

Also, for different types of approximating distributions the probabilities for missing fragments are calculated. Based on these results, we conclude that at least two missing fragments of mass 500-1000 g should exist (e.g. for the here referred Weibull distribution, this probability is around 0.839).

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