A Meteor Propagation Model Based on Fitting the Differential Equations of Meteor Motion

Pete Gural

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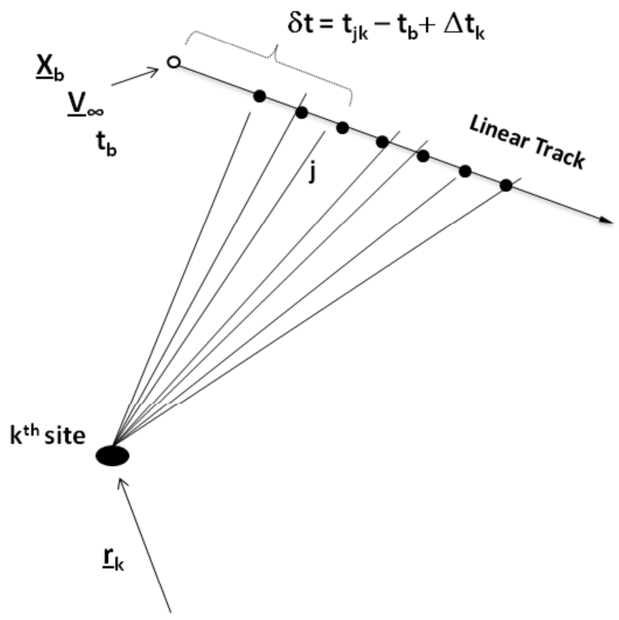
peter.s.gural@saic.com

Goal and Existing Models

Fully-Coupled Multi-Camera Trajectory Estimation Requires a Propagation Model*

Single k^{th} Camera Motion Model:

$$\underline{x}_k(t=j\delta_s) = \underline{x}_b + \underline{V}_\infty * [t + \Delta t_k - t_b] + a(t + \Delta t_k - t_b)$$



Published Propagation Models

$$x(t) = x_o + V_e t$$

$$x(t) = x_o + V_e t + a t^2$$

$$x(t) = x_o + V_e t + C e^{kt}$$

Whipple F. L., and Jacchia L. G. (1957) "Reduction methods for photographic meteor trails." Smithsonian Contributions to Astrophysics 1, pp. 183-206

Does not fit a deeply penetrating fireball over the ENTIRE trajectory at once

** Gural, P (2012) "A new method of meteor trajectory determination applied to multiple unsynchronized video cameras" Meteoritics & Planetary Science ??? pp. 1-14. Also see IMC 2011 PPT presentation.*

Dynamical Equations of Meteor Motion

Drag: $$M\frac{dV}{dt} = -\frac{1}{2}c_d \rho_a V^2 S,$$

Straight Line Propagation: $$\frac{dh}{dt} = -V \sin\gamma,$$

Mass Loss: $$H^* \frac{dM}{dt} = -\frac{1}{2}c_h \rho_a V^3 S$$

Assumptions		*Dimensionless Parameters*	
Isothermal Atmosphere:	$\rho = \exp(-y)$	Ballistic Coefficient:	$\alpha = \frac{1}{2}c_d \frac{\rho_0 h_0 S_e}{M_e \sin\gamma},$
Mass – Shape Power Law:	$\frac{S}{S_e} = (\frac{M}{M_e})^{\mu}$	Mass Loss Parameter:	$\beta = (1-\mu)\frac{c_h V_e^2}{2c_d H^*}$
		Power Law (rotation?):	$\mu = \log_m s$

From "Fireball Aerodynamics and Luminosity" PDF by Gritsevich and Koschny, IMC 2010

Solutions and Equation for dV / dt

Mass as a function of Velocity: $$m(v) = \exp\left(-\beta \frac{1-v^2}{1-\mu}\right)$$

Normalized Variables

Height as a function of Velocity: $$y(v) = \ln 2\alpha + \beta - \ln(\overline{E}i(\beta) - \overline{E}i(\beta v^2))$$

Exponential Integral: $$\overline{\mathrm{E}}\mathrm{i}(x) = \int_{-\infty}^{x} \frac{e^z dz}{z}$$

Acceleration

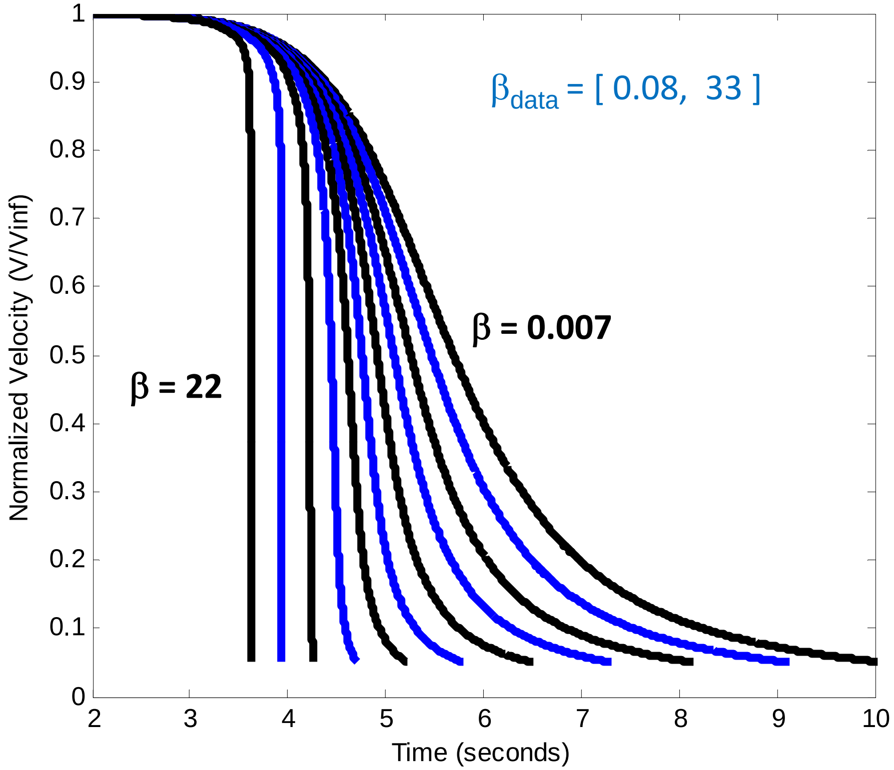
$$d\nu / dt = (-V_e \sin\gamma / 2h_o)\ \nu^2\ \exp(-\beta\nu^2)\ [\ \bar{E}i(\beta) - \bar{E}i(\beta\nu^2)\ ]$$

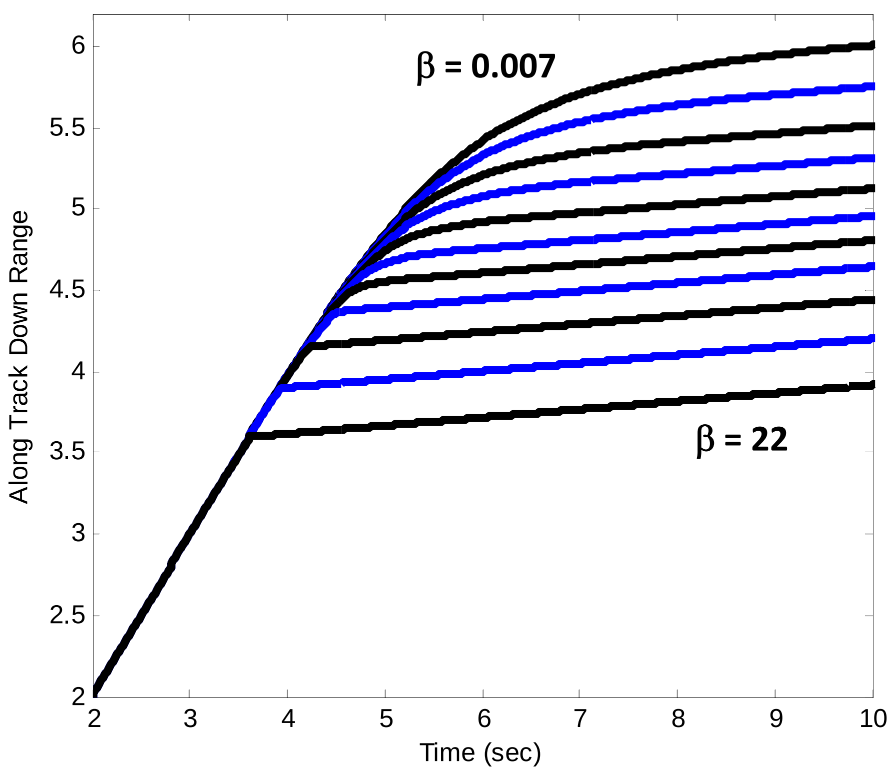
Function ONLY of a scale factor and β ! → How to obtain x(t, β) ?

Velocity and Position Profiles

$$t = (-2\, h_o / V_e \sin\gamma) \int_1^{\nu} d\nu \;\; \nu^{-2} \exp(\beta\nu^2) / [\, \bar{E}i(\beta) - \bar{E}i(\beta\nu^2) \,]$$

Velocity $\nu(t,\beta)$ $\int dt \rightarrow$ **Position $x(t,\beta)$**





β = 22.003 12.603 8.132 5.896 4.669 3.797 2.989 2.222 1.519 0.755 0.007

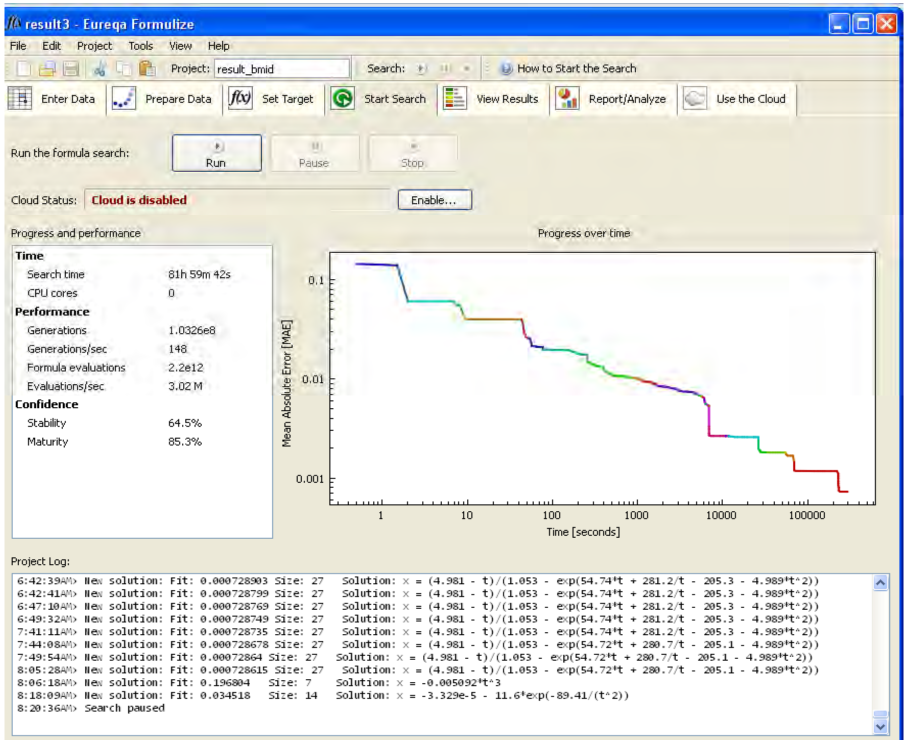
Innisfree

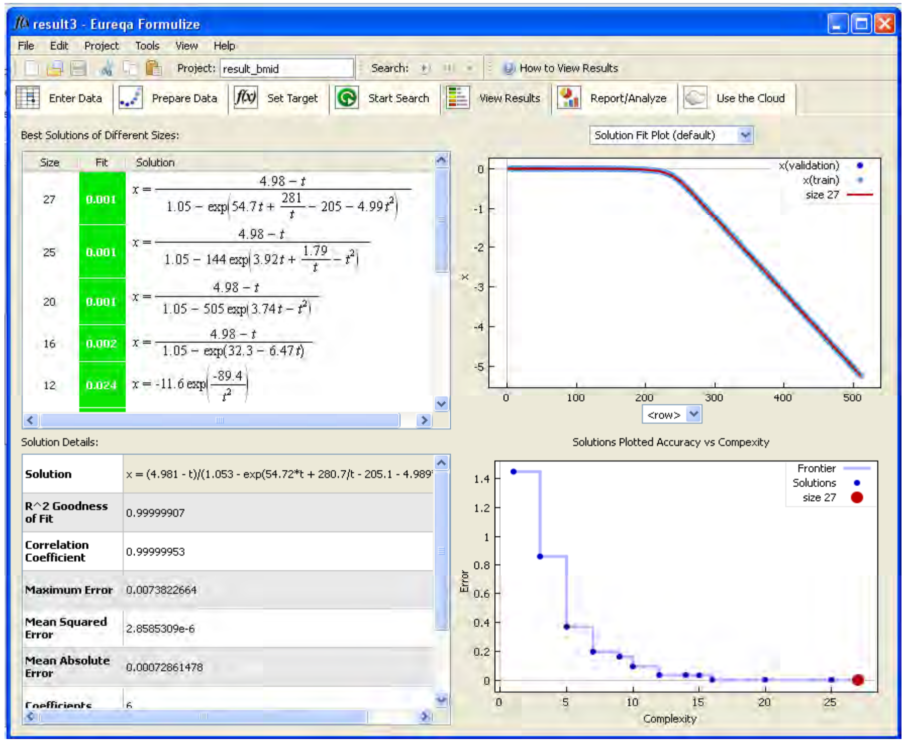
Eureqa Formulize

"A software tool for detecting equations and hidden mathematical relationships in your data"

Enter Data	Prepare Data	Set Target	Start Search	View Results	Analysis Tools	Use The Cloud

$$f(x) = \quad + \quad - \quad \times \quad \div \quad \sin x \quad \cos^{-1} x \quad \tanh x \quad e^x \quad \log x \quad ! \quad y^x \quad \surd \quad ...$$





"Best" Model Found – Thus Far

$$x(t) = t + (c_1 + c_2 t) / \{ c_3 + c_4 \exp[(c_5 + c_6 t) / (c_7 + \exp(t)) + c_8 t] \}$$

$c_1 = +0.34087 + 2.6517\,\beta$

$c_2 = -0.50104 - 2.6621\,\beta + 0.89817\,\beta^2 - 0.097926\,\beta^3$

$c_3 = +0.81913 + 4.2046\,\beta - 1.5428\,\beta^2 + 0.16662\,\beta^3$

$c_4 = +19.548 + 109.15\,\beta - 41.522\,\beta^2 + 4.0013\,\beta^3$

$c_5 = +179.04 + 425.61\,\beta - 65.534\,\beta^2$

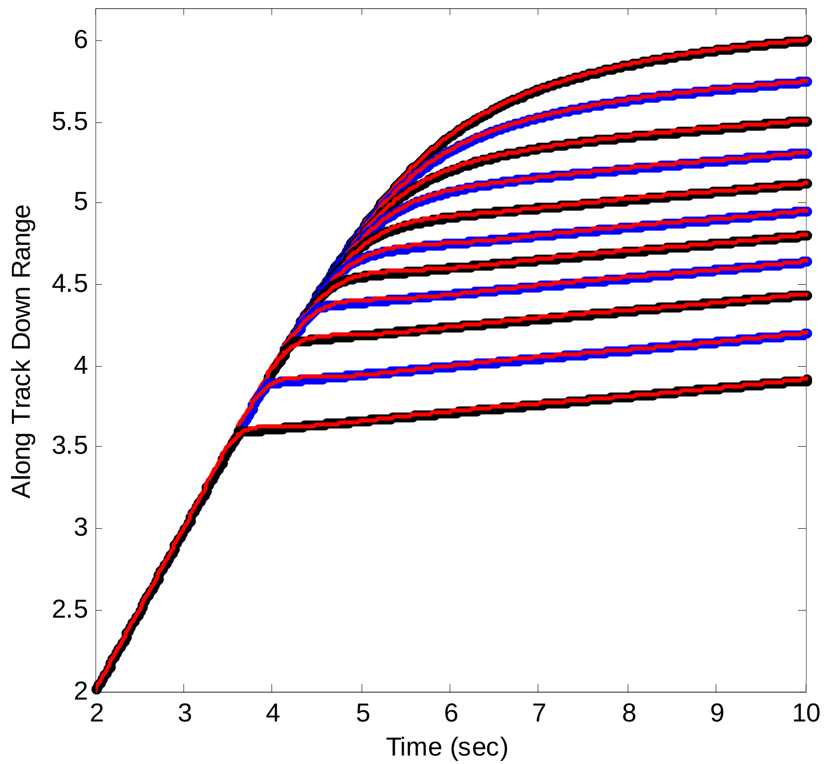
$c_6 = +28.46 - 77.083\,\beta + 9.1122\,\beta^2$

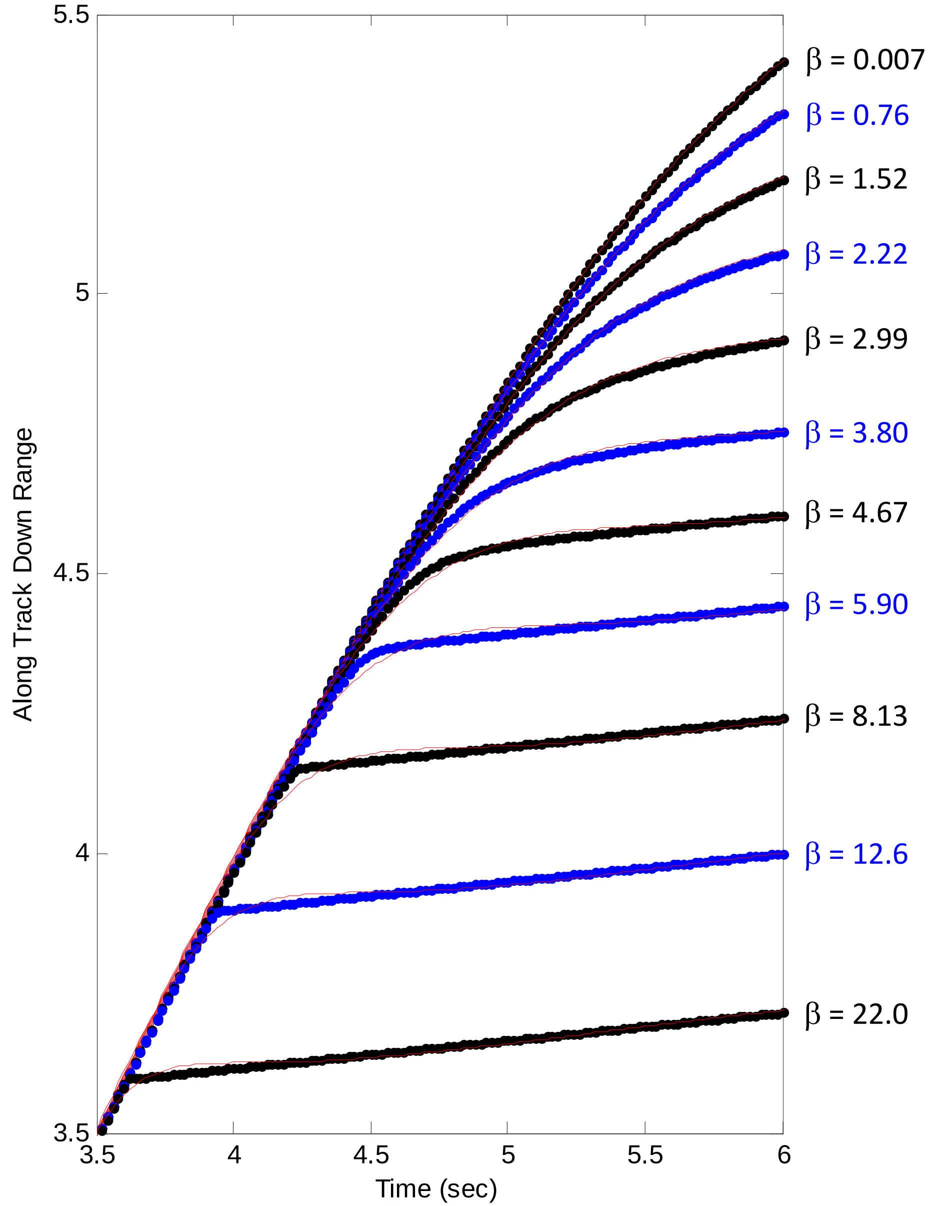
$c_7 = +45.027 + 129.66\,\beta - 228.04\,\beta^2 + 223.16\,\beta^3 - 124.53\,\beta^4$
$+ 39.119\,\beta^5 - 6.4418\,\beta^6 + 0.42978\,\beta^7$

$c_8 = +0.40731 + 0.033279\,\beta$

$\beta < 3$

Performance Fit Results





80% of Fireballs had $\beta < 3$

Next Steps

- **Find a model valid for high mass loss coef $\beta > 3$**
  - *Continue with Eureqa Formulize, use other functions*
  - *Remove the terminal velocity (dark flight) portion of data → Eureqa*
  - *Possibly merge two models*
- **Test on actual fireball tracks**
- **Incorporate into the multi-parameter fit trajectory code**
  - *Add new propagation model(s)*
  - *Errors per measurement & associated cost function*
  - *Switch from simplex to simulated-annealing minimization*

Assuming I am not too distracted by Spectral-CAMS !