Modern models of fracture meteoroids

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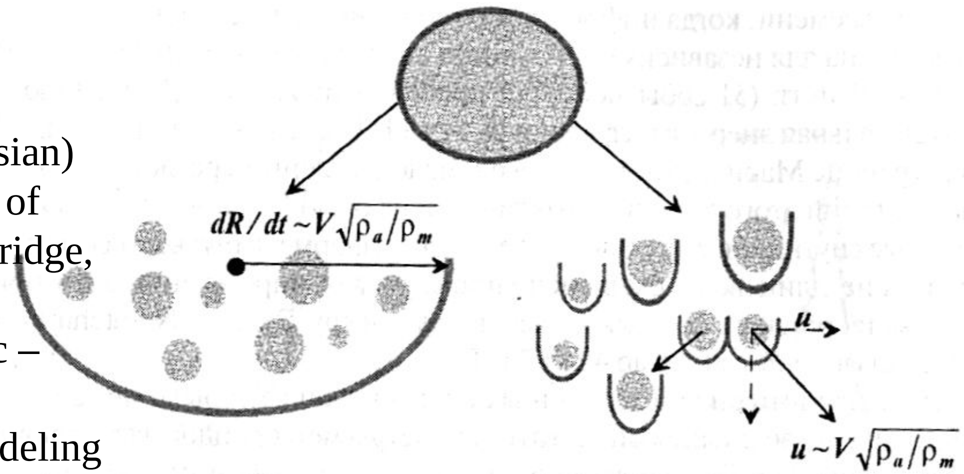
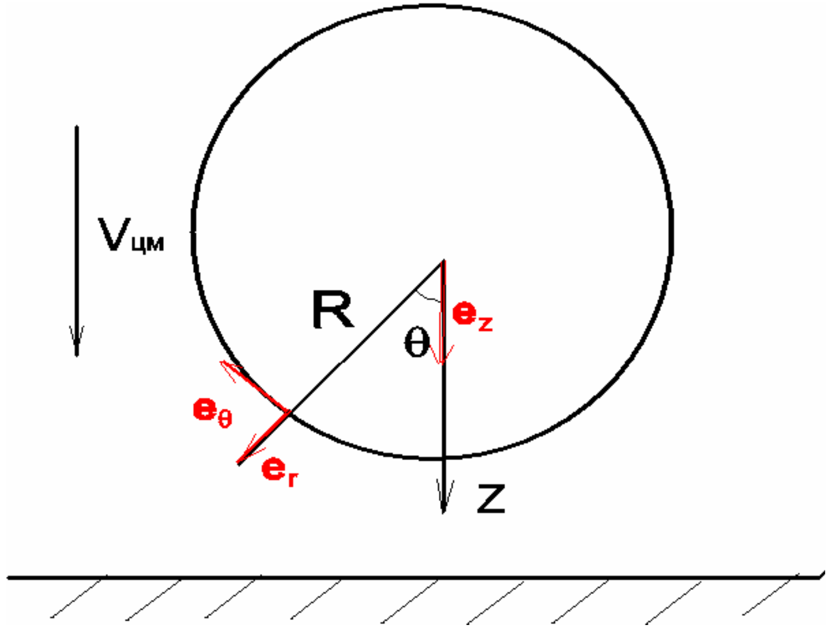


Elastic theory equation set

$$
\begin{cases}
\dfrac{1}{1-2\nu}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \Delta\mathbf{u} = -\dfrac{3}{8}\left(\alpha - \dfrac{3}{8G}\beta\right)\rho V^2 \dfrac{1}{R}\mathbf{k} \\
\sigma_r = \sigma(\theta), \quad if \;\; r = R \\
\tau_{r\theta} = \tau(\theta), \quad if \;\; r = R
\end{cases}
$$



$$
\sigma_r = \sigma(\theta) = \begin{cases}
-\alpha\rho V^2 \cos^2\theta, \; 0 \le \theta \le \dfrac{\pi}{2} \\
0, \quad \dfrac{\pi}{2} \le \theta \le \pi
\end{cases}
$$

$$
\tau_{r\theta} = \tau(\theta) = \begin{cases}
-\beta\rho V^2 \cos\theta \sin\theta, \; 0 \le \theta \le \dfrac{\pi}{2} \\
0, \quad \dfrac{\pi}{2} \le \theta \le \pi
\end{cases}
$$

ν - Poisson coefficients

r, θ - spherical coordinates

k –flow direction unit vector

Solution:

$$u_r = -r^2 \frac{1-2\nu}{1+\nu} P_1(\theta) \frac{3}{8}(\alpha+\beta)\frac{\rho_g V^2}{GR} - \frac{\alpha\rho_g V^2}{4G} R \left\{ \frac{1-2\nu}{3(1+\nu)} x + \frac{\beta}{\alpha}\frac{3}{8}\frac{4\nu-1}{1+\nu} x^2 P_1(\theta) + \right.$$

$$+\left[-\frac{\left(1-\frac{\beta}{\alpha}\right)4\nu}{7+5\nu} x^3 + \frac{7+2\nu+3\frac{\beta}{\alpha}}{3(7+5\nu)} 2x \right] p_2(\theta) + \left[-\frac{7(1+4\nu)\left(1-\frac{3}{2}\frac{\beta}{\alpha}\right)}{12(13+7\nu)} x^4 + \frac{7\left(7+\nu-2(2-\nu)\frac{\beta}{\alpha}\right)}{16(13+7\nu)} x^2 \right] P_3(\theta) +$$

$$\left. +\left[\frac{11\left(1-\frac{5\beta}{2\alpha}\right)(3+4\nu)}{64(31+11\nu)} x^6 - \frac{55\left(17+\nu-3(9-\nu)\frac{\beta}{\alpha}\right)}{786(31+11\nu)} x^4 \right] p_5(\theta) + \ldots - \right\}$$

$$u_\theta = -r^2 \frac{1-2\nu}{1+\nu} \frac{dP_1}{d\theta} \frac{3}{8}(\alpha+\beta)\frac{\rho_g V^2}{GR} - \frac{\alpha\rho_g V^2}{4G} R \left\{ \frac{3\beta}{16\alpha}\frac{(6-4\nu)}{(1+\nu)} x^2 \frac{dP_1}{d\theta} + \left[-\frac{\left(1-\frac{\beta}{\alpha}\right)(7-4\nu)}{3(7+5\nu)} x^3 + \frac{7+2\nu+3\frac{\beta}{\alpha}}{3(7+5\nu)} x \right] \frac{dP_2}{d\theta} + \right.$$

$$\left. +\left[-\frac{7(2-\nu)\left(1-\frac{3}{2}\frac{\beta}{\alpha}\right)}{12(13+7\nu)} x^4 + \frac{7\left(7+\nu-2(2-\nu)\frac{\beta}{\alpha}\right)}{48(13+7\nu)} x^2 \right] \frac{dP_3}{d\theta} + \left[\frac{11\left(1-\frac{5\beta}{2\alpha}\right)(5-2\nu)}{192(31+11\nu)} x^6 - \frac{11\left(17+\nu-3(9-\nu)\frac{\beta}{\alpha}\right)}{786(31+11\nu)} x^4 \right] \frac{dP_5}{d\theta} + \ldots \right\}$$

Где *G*- модуль сдвига, *x=r/R*, $P_1(\theta)$,…, $P_5(\theta)$ – Legendre polynoms

Stress state

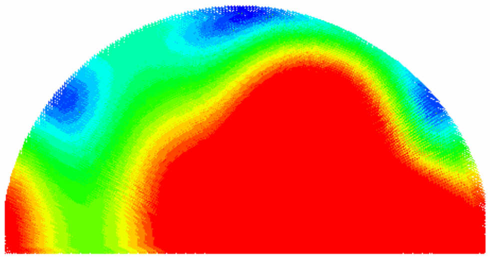
$$\varepsilon_{\alpha\beta} = \begin{cases} \varepsilon_r = \dfrac{\partial u_r}{\partial r} \\ \varepsilon_\theta = \dfrac{1}{r}\left(\dfrac{\partial u_\theta}{\partial \theta} + u_r\right) \\ \varepsilon_\varphi = \dfrac{1}{r}(u_\theta \operatorname{ctg}\theta + u_r) \\ \gamma_{r\theta} = \dfrac{1}{r}\dfrac{\partial u_r}{\partial \theta} + r\dfrac{\partial}{\partial r}\left(\dfrac{u_\theta}{r}\right) \end{cases}$$

$$\sigma_\alpha = 2G\left\{\varepsilon_\alpha + \frac{\nu\sum_{i=1}^{3}\varepsilon_i}{1-2\nu}\right\}$$

$$\tau_{\alpha\beta} = G\gamma_{\alpha\beta}\ ; \qquad \alpha,\beta = r,\theta,\varphi, \qquad \alpha \neq \beta$$

Stress intensity

$$\tau_i = \sqrt{I_2} = \sqrt{\frac{1}{6}\left[(\sigma_r-\sigma_\theta)^2 + (\sigma_r-\sigma_\varphi)^2 + (\sigma_\theta-\sigma_\varphi)^2 + \tau_{r\theta}^2\right]}$$



Thermoelastic stress

$$T(r,t) = \frac{2RT_0}{\pi r}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{\pi n r}{R}\,e^{-\kappa n^2\pi^2 t/R^2}$$

$$\bar{T}(r,t) = 6T_0\left(\frac{R_0}{\pi r}\right)^2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(\cos\frac{\pi n r}{R} - \frac{R}{n\pi r}\sin\frac{\pi n r}{R}\right)e^{-\kappa n^2\pi^2 t/R^2}$$

$$\text{где}\quad \bar{T}(r,t) = \frac{3}{r^3}\int_0^r x^2 T(x,t)\,dx$$

$$\frac{a}{r}\cdot\frac{\partial^2(rT)}{\partial r^2} = \frac{\partial T}{\partial t}$$

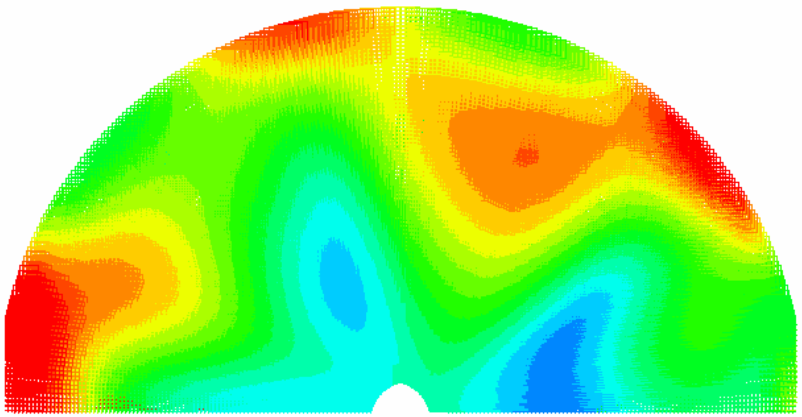
$$T\big|_{t=0} = 0$$

$$T\big|_{r=R} = T^*$$

$$\sigma_{rr} = \frac{2}{3}\frac{E\kappa}{1-\nu}\left[\bar{T}(R,t) - \bar{T}(r,t)\right]$$

$$\sigma_{\varphi\varphi} = \frac{1}{3}\frac{E\kappa}{1-\nu}\left[2\bar{T}(R,t) + \bar{T}(r,T) - 3T(r,t)\right] = \sigma_{\theta\theta}$$

a – thermo conductivity coefficient
α_T – coefficient of thermal expansion
E=2G(1+ν) – Young's modulus
ν – Poisson ratio



Elastic and thermoelastic stress (for a small body)

Criteria of elastic fragmentation

Condition of no rising flow strength

$$\frac{d}{dt}\left(\rho V^2\right) \le 0, \quad \text{где} \quad \rho = \rho_0 \exp\left(-\frac{h - V_* t + at^2/2}{H}\right)$$

$$V(t) = V - at, \quad a = \frac{3V_*^2 \rho_0}{8r\rho_T} \exp\left(-\frac{h_*}{H}\right)$$

$$r \le \frac{3}{4}\frac{H\rho_0}{\rho_T} e^{-\frac{h_*}{H}}$$

No fragmentation occur

due to elastic stress

$$h_* = -H \ln \frac{\sigma_* 2\sqrt{2}}{\alpha \rho_0 V^2 \sqrt{\Sigma_{\max}}}$$

The condition of safety: the dense of shape change potential energy less then critical one

$$\sqrt{3}\tau_i = \sqrt{3}\sqrt{I_2} < \sigma_*$$

If $V \cong const$, then $I_2 = I_2(\rho_г)$

$$\frac{\rho_г(h)}{\rho_г(H)} = e^{-\frac{h}{H}}$$

$$\frac{\sqrt{3}\sqrt{I_2(h)}}{\sqrt{3}\sqrt{I_2(H)}} = \frac{\sigma^*}{\sqrt{3}\sqrt{I_2(H)}} = \frac{\rho_г(h)}{\rho_г(H)} = e^{-\frac{h}{H}}$$

$$h = -H \ln \frac{\sigma_*}{\sqrt{3}\left(\sqrt{I_2(H)}\right)_{max}}$$

$$h = -H \ln \frac{\sigma_* 2\sqrt{2}}{\alpha \rho_H V^2 \left(\sqrt{\Sigma_{\max}}\right)}$$

$$\sigma_* = 700\ atm, \quad V = 30\,km/s, \quad h_{start} = 7{,}868\,km$$

Falls to the surface

$$\sigma_* = 50\ atm, \quad V = 30\,km/s, \quad h_{start} = 37{,}6\,km, \quad h_{end} = 26{,}6\,km$$

Effect of heat explosion

$$\frac{dN_m}{dm} = Cm^{\frac{k}{3}-2}, k = 1.2$$

Spectrum of particle size

N_m – number of paricles of mass m, C =const

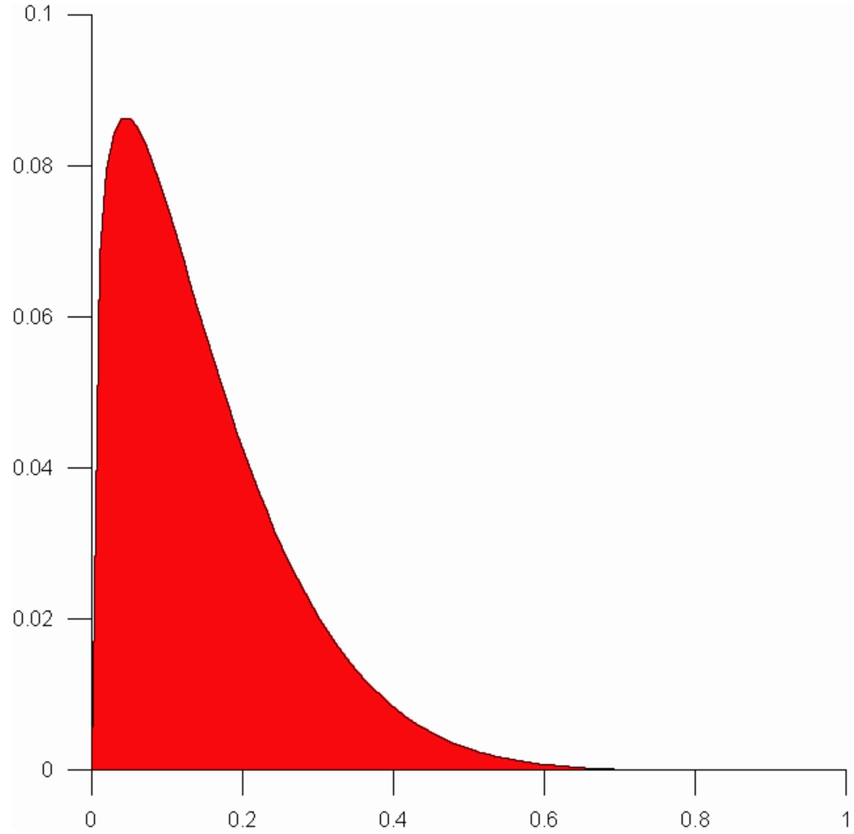
$$i^* \frac{dm}{dt} = \frac{1}{2} c_H \rho_g V^3 S$$ ablation rate

Solution

$$t_* = \frac{2 i^* \rho_b}{c_H \rho_g V^3} R$$

$$l = \frac{4}{3}\frac{\rho_b}{\rho_g}\left[-R - \left(\frac{\gamma}{V_\infty} + R\right)\ln\left(\frac{\gamma/V_\infty}{\gamma/V_\infty + R}\right)\right],$$

$$\gamma = \frac{2}{3}\frac{\rho_T}{\rho_2 i_*} R^2$$



$$\frac{dM(t)}{dt} = \int_0^1 N(\bar{m}) \frac{dm}{dt} d\bar{m}$$

$$I = -\tau \frac{V^2}{2} \frac{dM}{dt}$$

Conclusions

The body moving in the planet atmosphere is under the influence of the aerodynamic loads, the forces of inertia and the heat flux. As a result, the body undergoes ablation and even could be completely destroyed.

- **First of all, the stressed state within the body at any time is determined through an accurate solution of the Lamé equations. Based on the obtained solution one can investigate the nature of the body destruction and evaluate the distraction altitude for well-known meteoroids if their composition and space velocities are known.**
- **During the flight of small fragments the thermo elastic forces become significant. Unlike large fragments the smaller ones warm in a short time resulting in accelerating the process of destruction, which also contributes to the ablation, i.e., reduce the fragments size.**
- **Finally, «thermal explosion» due to the rapid evaporation of small fragments cloud with a typical range of sizes of fragments was considered. Assessment of the length of run and time of evaporation of small particles allows one to speak about the explosive outbreak and disappearance of the asteroid in the final stages of its collapse.**

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