

# Meteoroid flux densities from radio observations: the latest developments

Cis Verbeeck

# Outline

1. Introduction
2. Why calculate meteoroid flux densities?
3. Kaiser method
4. Belkovich method
5. Ryabova method
6. Verbeeck method
7. Summary

# 1. Introduction

- Meteoroid **flux density**  $Q(m_*)$ : number of stream meteoroids having masses  $\geq m_*$  intersecting unit area in unit time ( $\perp$  velocity vector)
- Typical values:  $Q(m_* = 10^{-3} \text{ g}) = 10^{-11}-10^{-12} \text{ m}^{-2} \text{ s}^{-1}$
- **Mass index  $s$** : exponent such that number  $dN_m$  of meteoroids having masses in  $[m, m + dm]$  is proportional to  $m^{-s}$
- Note that  $\frac{Q(m)}{Q(m_*)} = \left(\frac{m}{m_*}\right)^{1-s}$

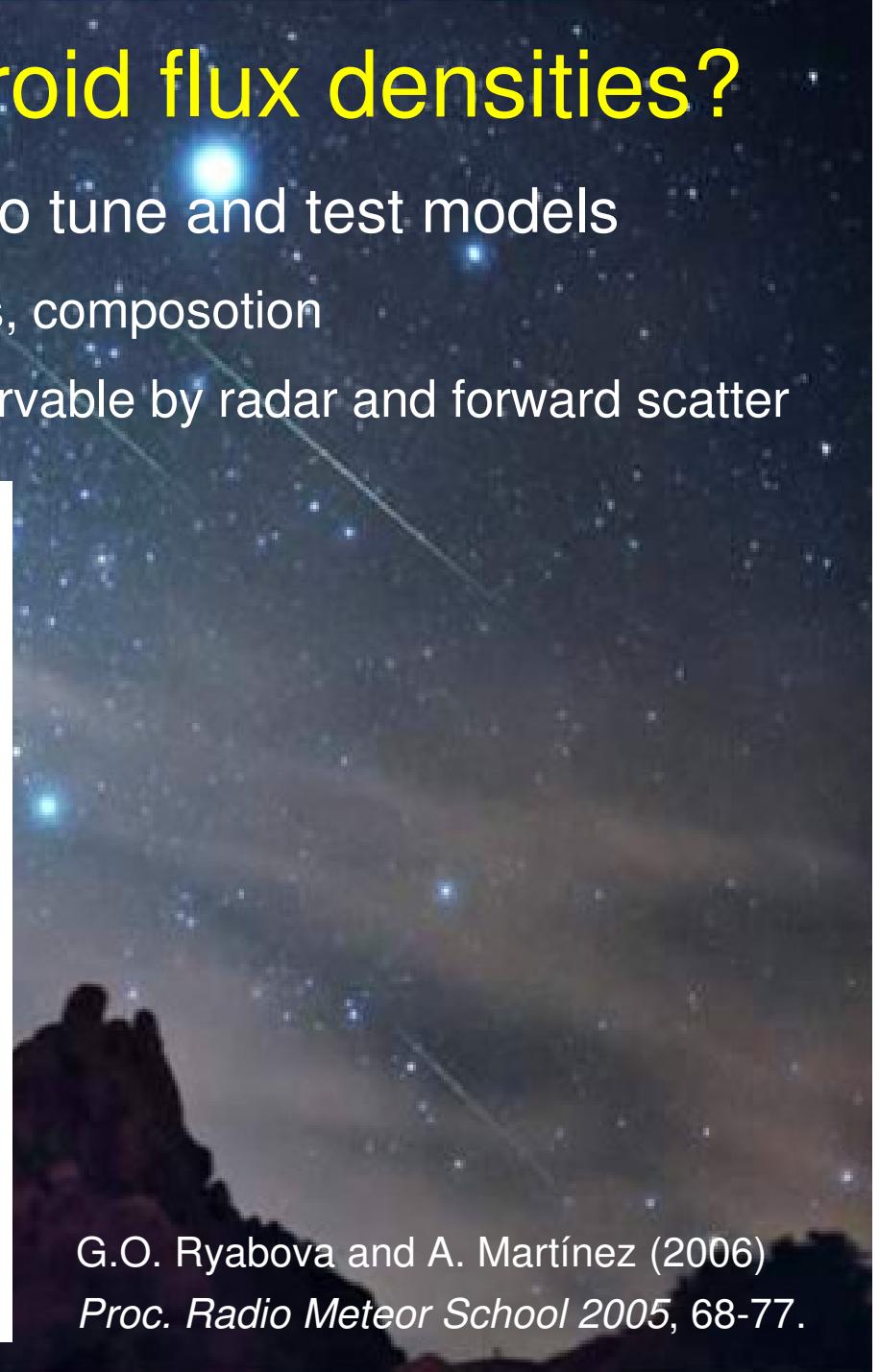
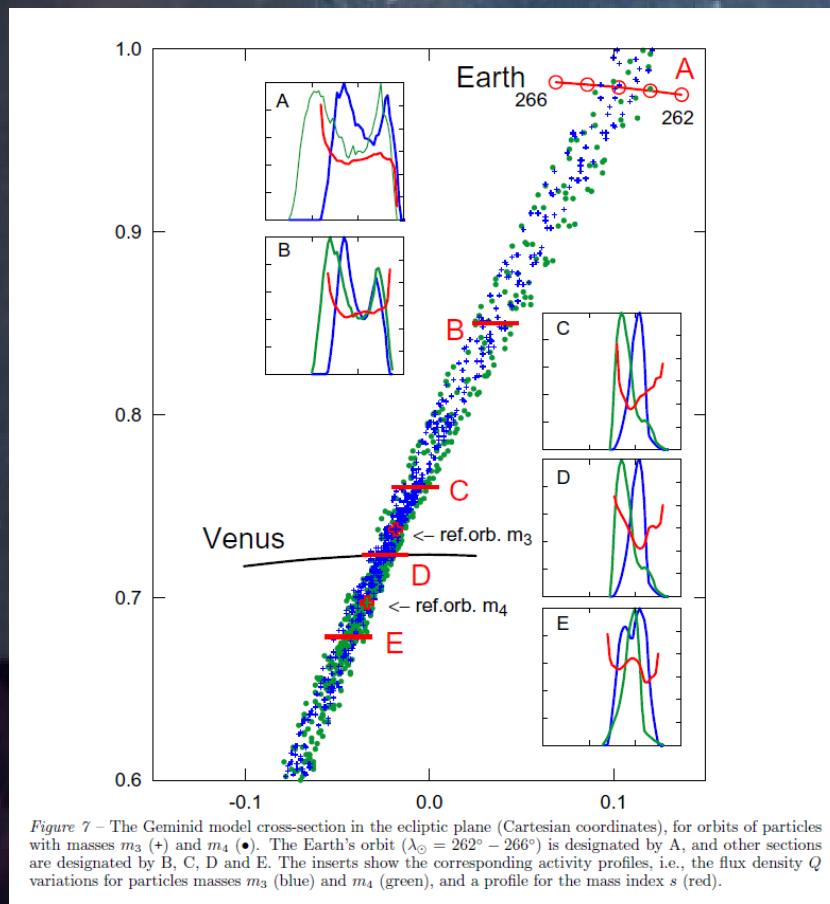
## 2. Why calculate meteoroid flux densities?

- a) Stream modeling: using flux density to tune and test models
- b) Meteoroid hazard for spaceflight
- c) Source of global particle layers in atmosphere

# Why calculate meteoroid flux densities?

## a) Stream modeling: using flux to tune and test models

- Solar System origin, dynamics, composition
- Smallest population only observable by radar and forward scatter



G.O. Ryabova and A. Martínez (2006)  
Proc. Radio Meteor School 2005, 68-77.

# Why calculate meteoroid flux densities?

b) Meteoroid hazard for spaceflight

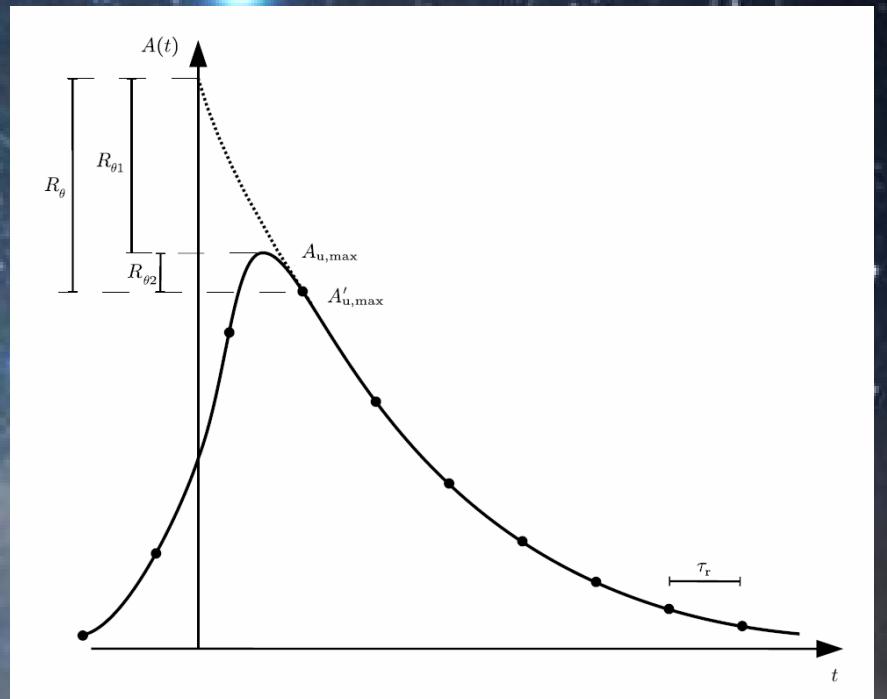
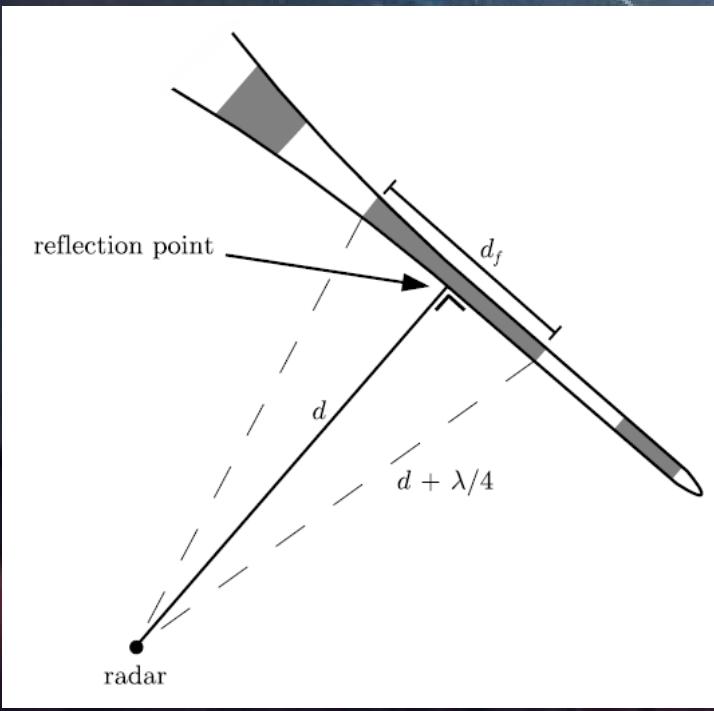
- 1999 Leonids: no Shuttle launches in November

c) Source of global particle layers in atmosphere

- Neutral metal atoms
- Metal ions (Sporadic E)
- Meteoric smoke particles (PMSE, NLC)

# Selection effects

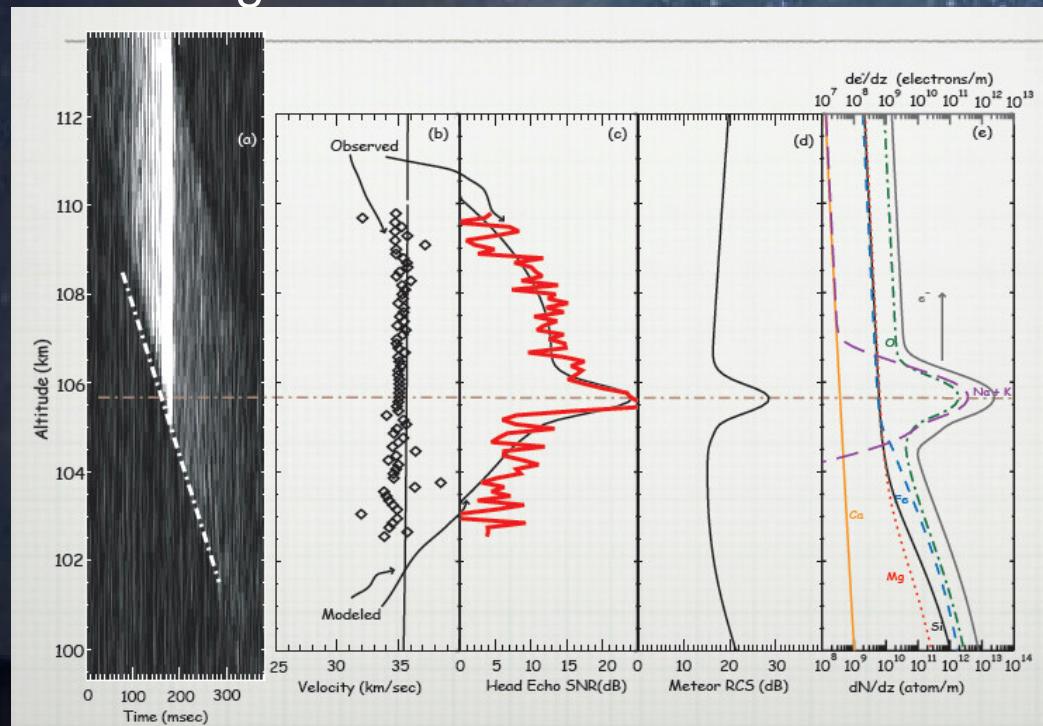
- Initial trail radius effect
  - ⇒ less high meteors observed
  - ⇒ flux density underestimated
- Finite velocity effect
  - ⇒ less slow meteors observed
  - ⇒ flux density underestimated



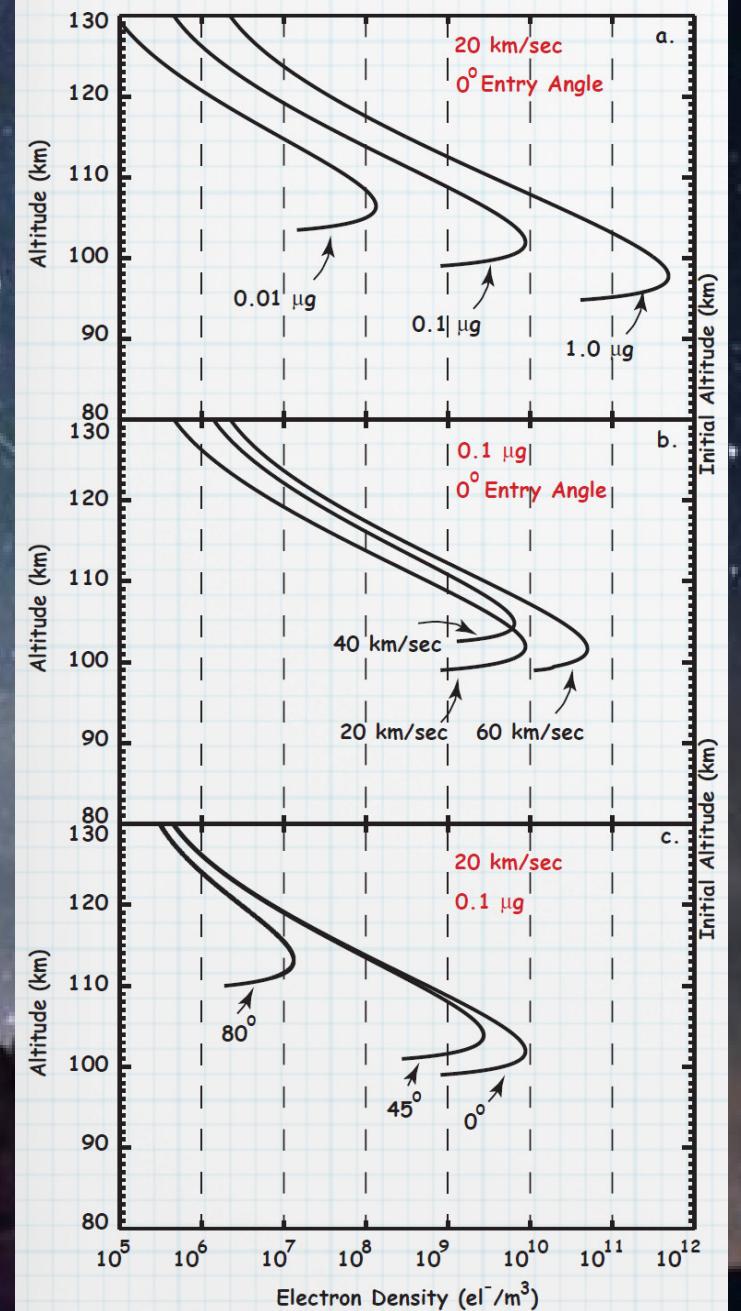
- PRF effect
  - ⇒ less short-lived meteors observed
  - ⇒ flux density underestimated
- Lower mass limit mismatch

# Ionization profiles

Mass (, velocity, zenith angle, composition)  
 $\Rightarrow$  electron line density  $a = a(h)$ ,  
 including start and end  
 height



D. Janches, L.P. Dyrud, S.L. Broadley, and J.M.C. Plane (2009) *Geophys. Res. Lett.* 36, L06101.



J.T. Fenzke and D. Janches (2008) *JGR* 113, A03304.

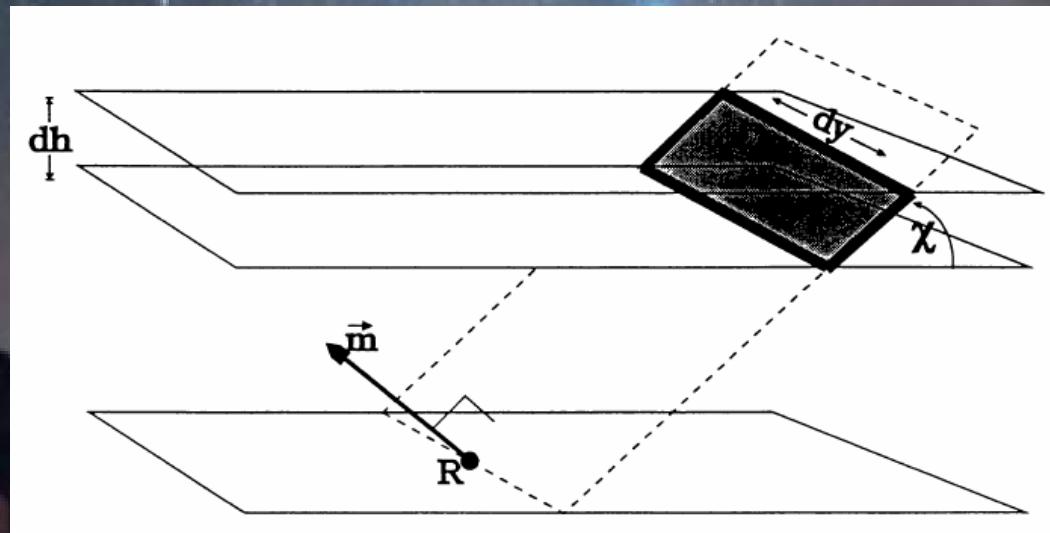
### 3. Kaiser method

- a) Basic ideas
- b) Selection effects

T. Kaiser (1960) *MNRAS* 121, 284-298.

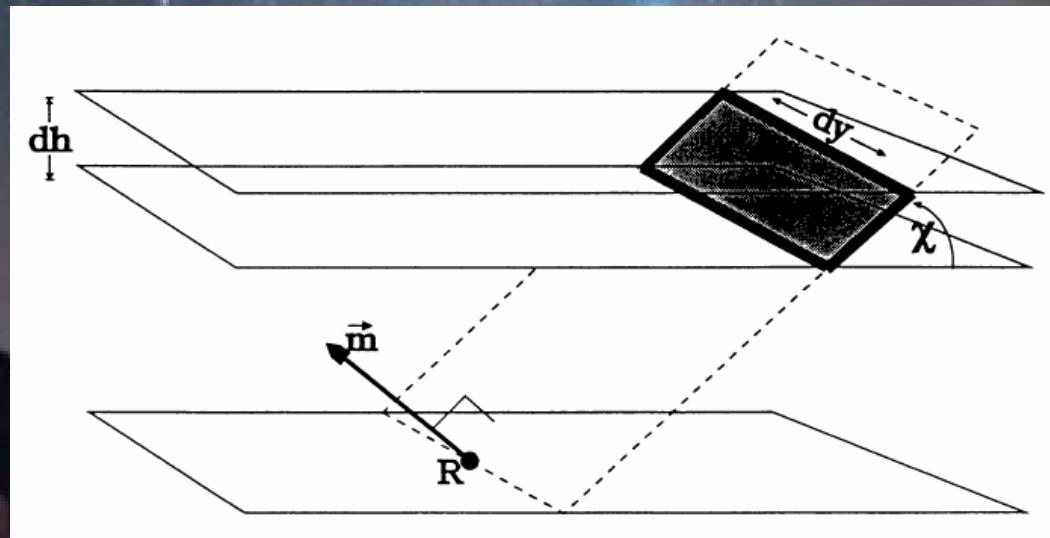
# Kaiser method: basic ideas (1)

- Count number  $N$  of meteoroids from particular radiant in particular period of time
- Fixed meteor height
- Fixed vertical meteor zone extent depending on stream velocity, entry angle and mass index
- Effective collecting area  $S = \text{vertical width} \cdot \text{horizontal « length »}$



# Kaiser method: basic ideas (2)

- Define  $m_0$  as minimal detectable mass in direction of max gain
- Horizontal « length » w.r.t.  $m_0$ :  $\sim (\cos \chi)^{s-1} \cdot \int (G_T G_R / R^3)^{(s-1)/2} dy$
- Flux density w.r.t.  $m_0$ :  $Q(m_0) = N/S$
- Wanted flux density:  $Q(m_*) = Q(m_0) \cdot (m_*/m_0)^{1-s}$



# Kaiser method: selection effects

- Initial trail radius effect
  - meteor height zone not dependent on signal strength
- Finite velocity effect
- PRF effect
- Lower mass limit mismatch

## 4. Belkovich method

- a) Basic ideas
- b) Extra features
- c) Results
- d) Selection effects

O.I. Belkovich (1971) *Statistical theory of meteor radar observations*. Kazan University, Russia (in Russian).

O.I. Belkovich and V.S. Tokhtasev (1974) *Bull. Astron. Inst. Czech.* 25:2, 112-115; 25:6, 370-374.

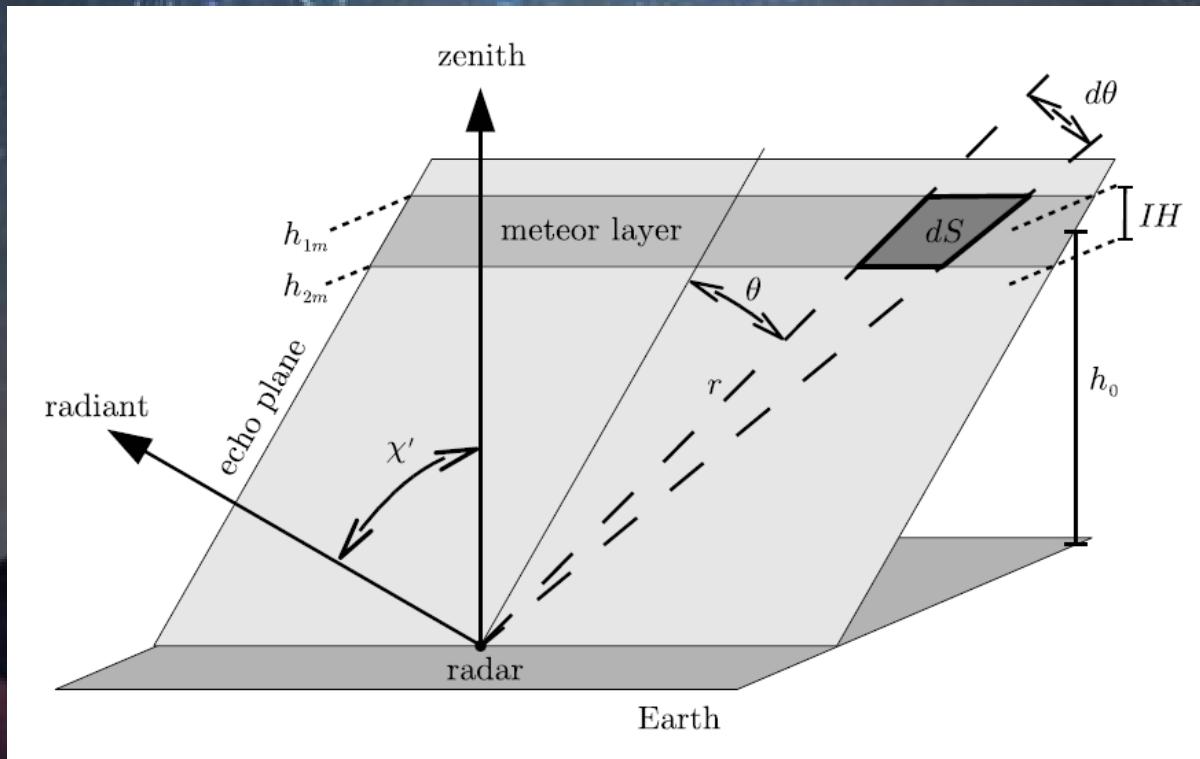
O.I. Belkovich and C. Verbeeck (2006) *Proc. Radio Meteor School 2005*, 38-47.

O.I. Belkovich and S. Suleymanova (1999). *Meteoroids 1998*, 103–106.

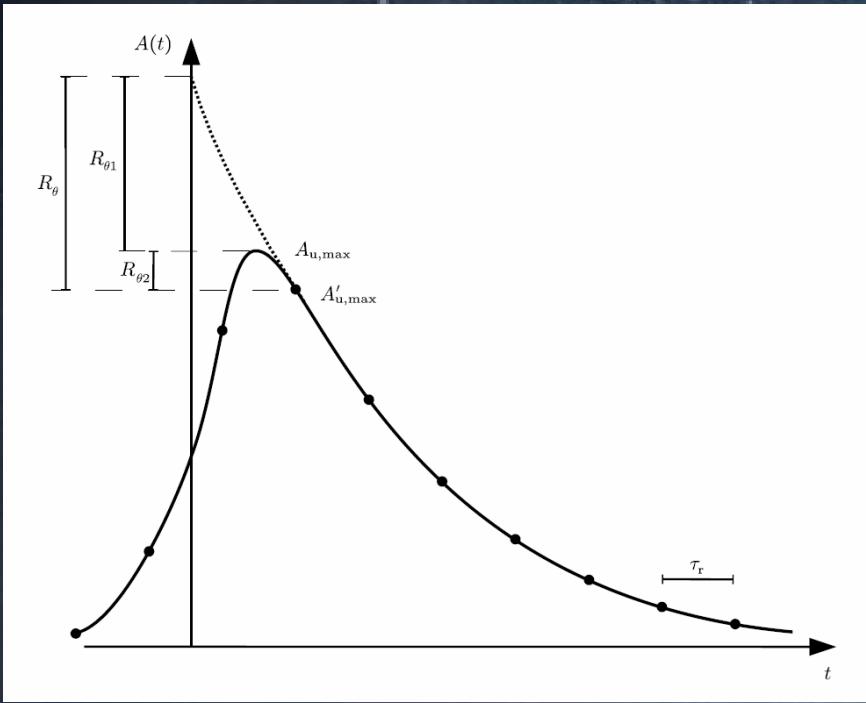
S. Suleymanova, C. Verbeeck, and J.-M. Wislez (2007) *Proc. IMC 2006*, 162-174.

# Belkovich method: basic ideas

- Similar to Kaiser technique, but using polar coordinates: partition meteor layer into trapezia  $dS_\theta$  defined by angles  $\theta$  and  $d\theta$
- $h_{1m}$  and  $h_{2m}$  dependent on mass (ionization profile)
- $dS_\theta =$  weighted average of trapezia for all masses above  $m_*$



# Belkovich method: extra features



- Finite velocity effect
- PRF effect
- Combined fine-tuning of mass index and flux density
- Meteor height zone dependent on (empirical) ionization profile
- Forward scatter version exists

The correction factor  $R_\theta$  statistically corrects for these two effects, and is given by:

$$R_\theta = \begin{cases} \exp(-0.79(kr_0)^{1.74} - 0.79\Theta_\theta^{0.88} + 0.14(kr_0)\Theta_\theta^{0.68}) & \text{if } \Theta_\theta < 1, \\ \left(\frac{1}{2\Theta_\theta}\right) \cdot \exp(-0.79(kr_0)^{1.74} + 0.14(kr_0) - 0.1) & \text{if } \Theta_\theta > 1, \end{cases} \quad (5)$$

where  $k = 2\pi/\lambda$ ,  $r_0$  is the initial radius, and  $\Theta_\theta$  a quantity quantifying the extent of the effect:

$$\Theta_\theta = \frac{\tau_f + \tau_r}{2\tau_0}, \quad (6)$$

where  $\tau_f$  is the time the meteoroid needs to travel half of the first Fresnel zone,  $\tau_r$  is the time between pulses, and  $\tau_0$  is the decay time at the characteristic height  $h_0$ .  $\Theta_\theta$  compares the duration of the two disturbing processes with the diffusion time constant, and thus indicates whether they are significant.

# Belkovich method: results

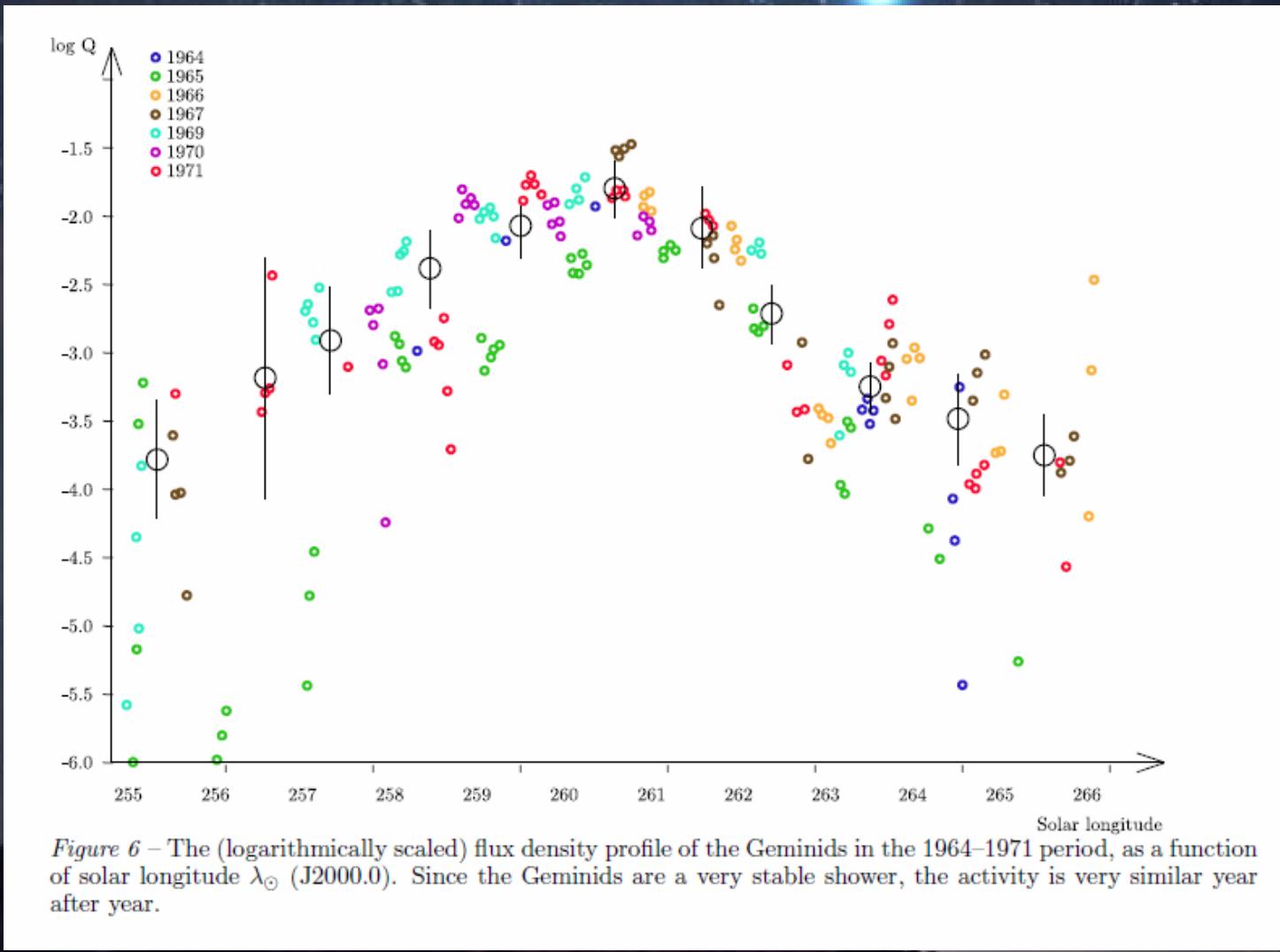


Figure 6 – The (logarithmically scaled) flux density profile of the Geminids in the 1964–1971 period, as a function of solar longitude  $\lambda_{\odot}$  (J2000.0). Since the Geminids are a very stable shower, the activity is very similar year after year.

# Belkovich method: selection effects

- Initial trail radius effect
  - meteor height zone for  $m$  not dependent on signal strength
- Finite velocity effect solved
- PRF effect solved
- Lower mass limit mismatch
  - lower mass limit varies throughout  $dS_\theta$

## 5. Ryabova method

- a) Basic ideas
- b) Sensitivity curves and ionization curves
- c) Selection effects

G.V. Andreev and G.O. Ryabova (1984) *Astronomiya i geodeziya* 10, 131-136;  
11, 22-30. (in Russian)

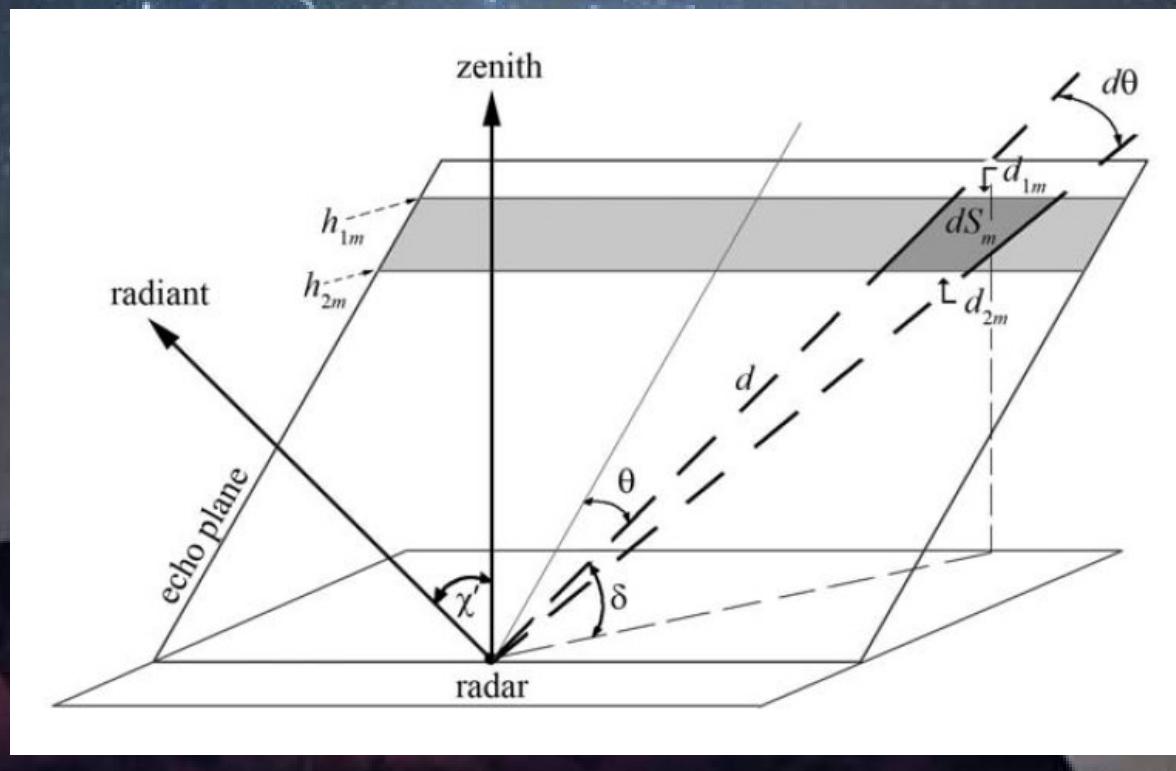
G.V. Andreev, A.E. Epishova, O.A. Mugruzina, and L.N. Rubtsov (1984) *Astronomiya i geodeziya* 13, 37-49. (in Russian)

G.O. Ryabova (2008) *WGN* 36:6, 120-123.

G.O. Ryabova (2009) *WGN* 37:2, 63-67.

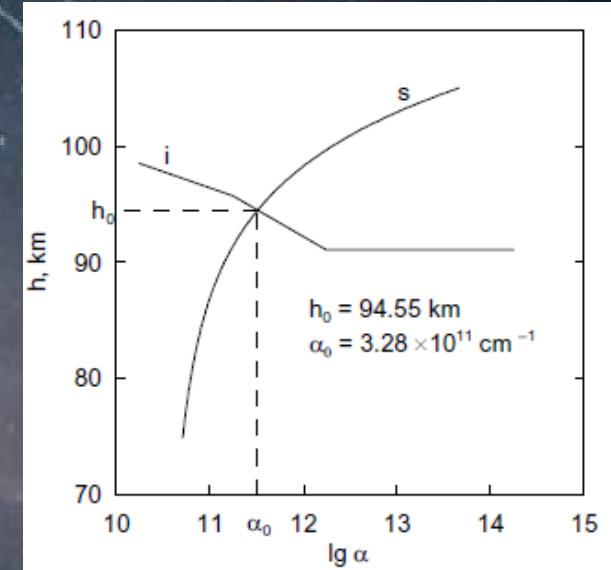
# Ryabova method: basic ideas

- $h_{1m}$  and  $h_{2m}$  correspond to detectable part of meteor trails (ionization profile and signal strength)  $\Rightarrow$  reliable ionization profiles needed
- $dS_\theta$  = weighted average of trapezia for all masses above  $m_\theta$
- To determine  $m_\theta$ , consider ionization profile and signal strength



# Ionization curves and sensitivity curves

- Ionization curve = physical model specifying for every line density  $\alpha$ , height  $h = h(\alpha)$  at which all meteors having  $\alpha$  as max line density, reach this max value  $\alpha$ .
- $\theta$  fixed. For every  $h$ , sensitivity curve specifies minimal detectable electron line density  $\alpha = \alpha(h)$  (corresponding to radar threshold).
- Ionization curve  $\cap$  sensitivity curve  $\Rightarrow$  unique  $h_\theta$  and  $\alpha_\theta$  such that meteor having max line density  $\alpha_\theta$  at  $h_\theta$  corresponds to threshold.
- $\alpha_\theta$  corresponds to  $m_\theta$ .



# Ryabova method: selection effects

- Initial trail radius effect solved
  - for every  $m$ , meteor height zone dependent on ionization profile and signal strength
- Finite velocity effect solved
- PRF effect solved
- Lower mass limit mismatch
  - lower mass limit varies throughout  $dS_\theta$



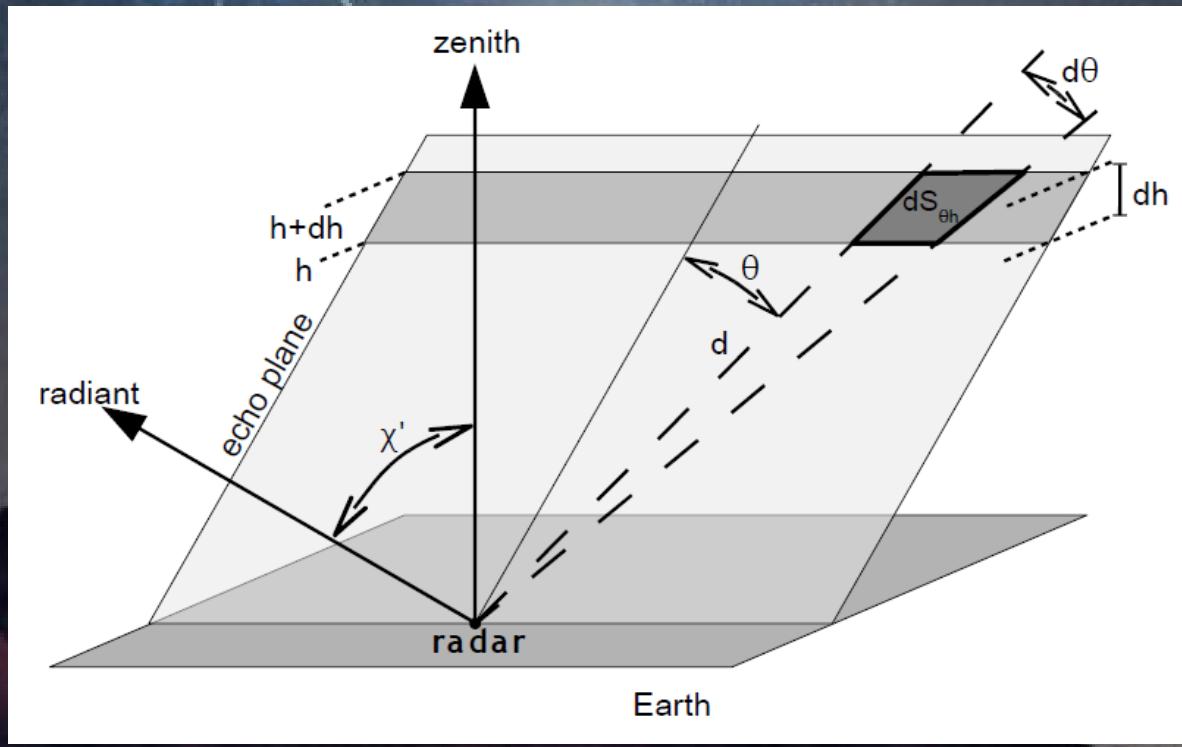
## 6. Verbeeck method

- a) Basic ideas
- b) Selection effects

# Verbeeck method: basic ideas

- For every  $\theta$  and  $h$ , determine min and max detectable masses  $m_{\theta h}$  and  $m'_{\theta h}$  (consider ionization profile and signal strength)
- Observed flux density in  $dS_{\theta h}$  = for masses between  $m_{\theta h}$  and  $m'_{\theta h}$

$$N = \frac{Q(m_*) \cdot m_*^{s-1}}{\sin^2 \chi'} \cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{I''(\theta)}{\cos^2 \theta} d\theta, \quad I''(\theta) = \int_{h_b}^{h_e} h \cdot (m_{\theta h}^{1-s} - m'_{\theta h}^{1-s}) dh.$$



# Verbeeck method: selection effects

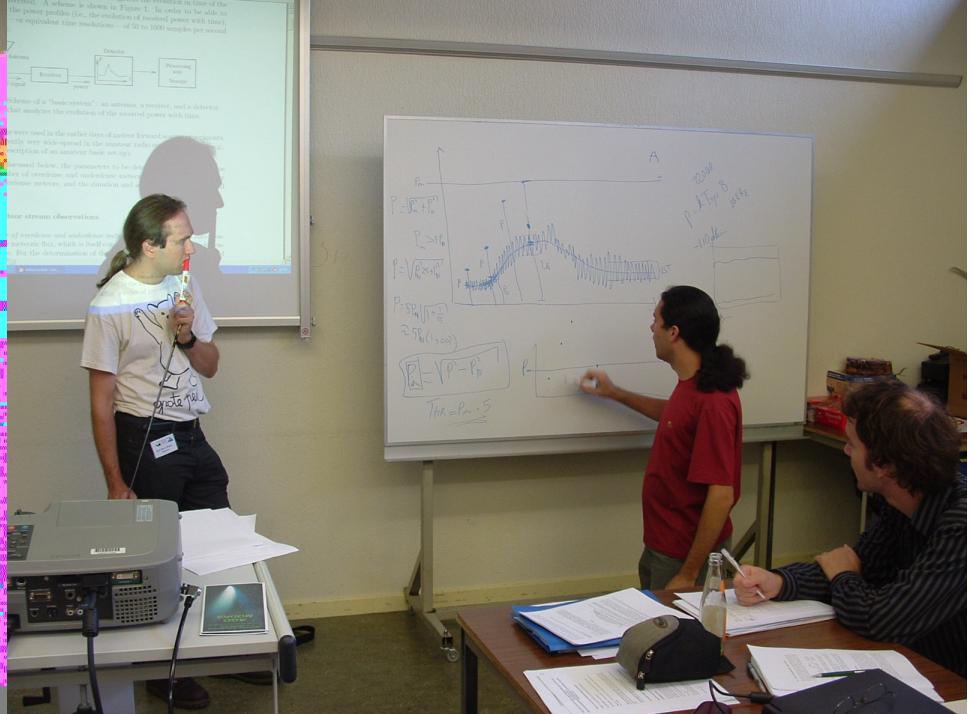
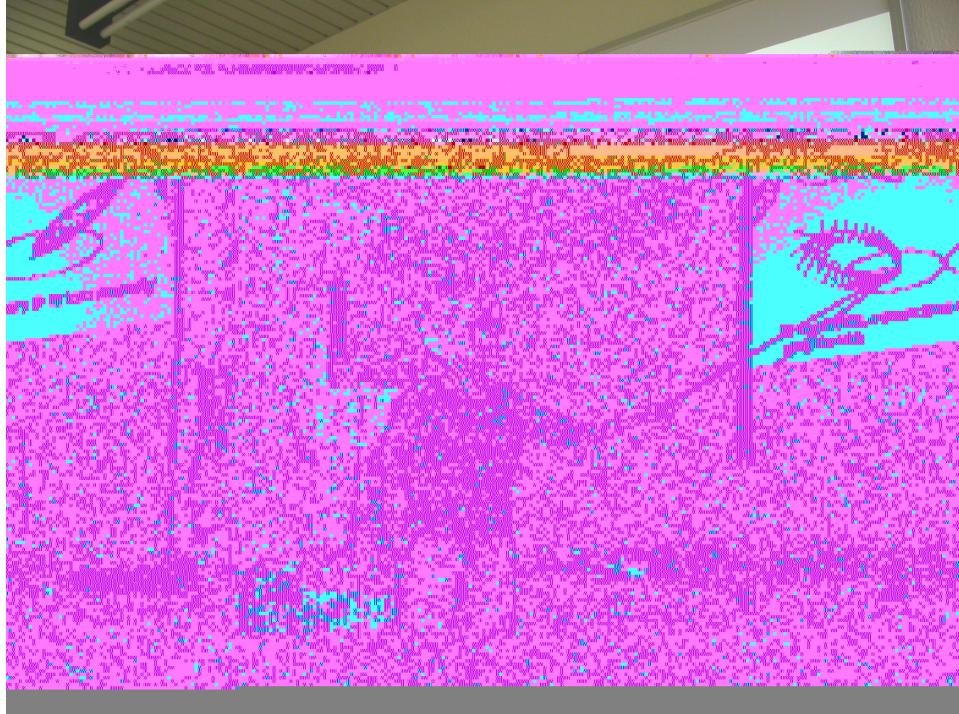
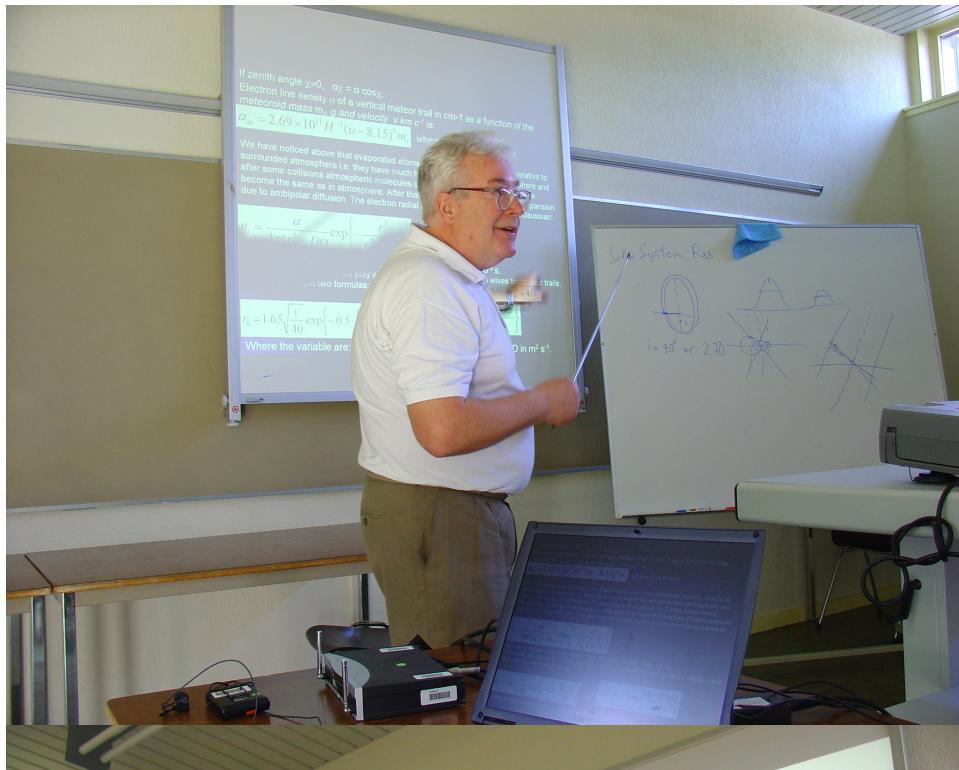
- Initial trail radius effect solved
  - for every  $\theta$  and  $h$ , meteor mass range dependent on ionization profile and signal strength
- Finite velocity effect solved
- PRF effect solved
- Lower mass limit mismatch solved
  - lower mass limit constant throughout  $dS_{\theta h}$

## 7. Summary

- Kaiser method suffers of several selection effects
- Belkovich method: finite velocity effect and PRF effect solved
- Ryabova method: + initial trail radius effect solved
- Verbeeck method: + lower mass limit mismatch solved
- Needed: accurate ionization profiles

# Acknowledgments

- Oleg Belkovich
- Galina Ryabova
- Svetlana Suleymanova
- Participants Radio Meteor Schools 2005-2006



12.  $G_T, G_R$  - are the transmitting and receiving antenna gains in the direction of the reflection point.

Now we have to go in the frame of references of the transmitter or receiver by the parallel shift and rotations of the excises:

$$x'_T = \cos \gamma_E \cdot (x + L) + \sin \gamma_E \cdot z'_E$$

$$y'_T = y,$$

$$z'_T = -\sin \gamma_E \cdot (x + L) + \cos \gamma_E \cdot z'_E$$

$$x'_R = \cos \gamma_E \cdot (x - L) - \sin \gamma_E \cdot z'_E$$

$$y'_R = y,$$

$$z'_R = \sin \gamma_E \cdot (x - L) + \cos \gamma_E \cdot z'_E$$

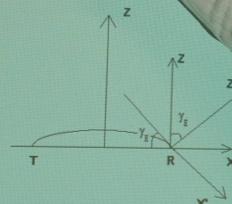
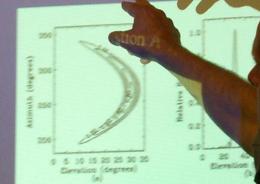
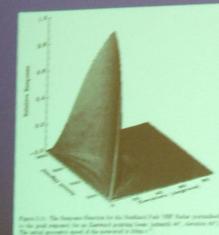


Figure 1.11: The Response Function for the Standard Park VHF Radar for a meteor with initial velocity  $v_0 = 10 \text{ km s}^{-1}$  and initial geometric spread  $\sigma = 10^\circ$ . The initial geometry is given by  $x_0 = 100 \text{ km}$ ,  $y_0 = 0$ ,  $z_0 = 0$ .



#### A: Back scatter systems :

The 'Response function' : System sensitivity (response) as function of position  $(Az, h)$ .



Figures 1.12: The Response Function for the Standard Park VHF Radar for a meteor with initial velocity  $v_0 = 10 \text{ km s}^{-1}$ . The initial geometry is given by  $x_0 = 100 \text{ km}$ ,  $y_0 = 0$ ,  $z_0 = 0$ . The initial geometric spread is  $\sigma = 10^\circ$ . In panel (a), the contours show the relative response as a function of the peak response as a function of elevation and azimuth. Panel (b) shows relative responses at an altitude of 2000 m.

#### B: Forward scatter systems : Hines 1955 : Cylindrical approximation

1958 : Ellipsoidal theory

Conditions for echo detection :

##### I. Geometrical (for specular reflections):

see PIA095 proc. pp. 46-48

##### II . Instrumental :

see PIA095 proc. pp. 59

1. The reflection point M must lie on the locus of T and R.
2. M must lie within the range of the instrument.
3. The meteor trail must be tangent to the Fresnel's ellipsoid

##### 4. Amplitude condition

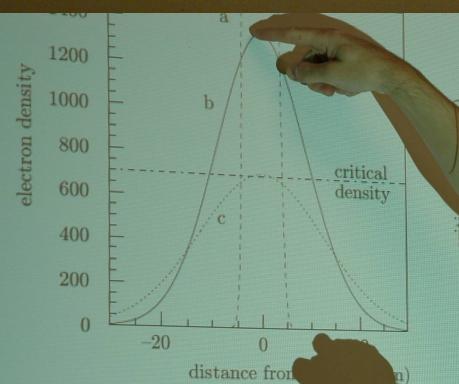


Figure 3 – Scattering off an overdense trail. The electron density line density  $n(r, t)$  (in  $\text{m}^{-3}$ ) is shown in panel (a). Panel (b) shows the evolution of the critical radius  $r_c$  of the trail (in m) as a function of the radial radius  $r$  (in m).

The maximal amplitude  $A_{o,\max}$  (in V) for the echo measured at the entrance of the radar receiver is (Kaiser, 1953):

$$A_{o,\max} = \alpha^{1/4} g_{02} F,$$

where  $g_{02}$  is the reflection coefficient for overdense trails, given by

$$g_{02} = 0.84 \cdot \left( \frac{\pi^2 r_e}{4e} \right)^{1/4},$$

where  $r_e$  is the radius of the electron (in cm),  $e$  is the charge of the electron, and  $\alpha$  is the reduced total cross-section of the trail.

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