Integration of mean orbits of meteoroid streams

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Plan

1. Traditional approach for mean orbit calculations
2. Jopek et al. method
3. Example of Neuschwanstein
4. Example of Quadrantids

Calculation of mean orbit

Typically one takes mean value of heliocentric orbital elements: a, e, i, ω, Ω.

Instead one can use: $1/a$, e, i, ω, Ω.
or: $1/a$, q, i, ω, Ω.
even: q, Q, i, ω, Ω can be used.

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Problem

Averaging kills important mathematical qualities:

- Simplest mathematical example:
  $(1+5)/2 = 3$
  $(1^2+5^2)/2 = 13$
  $3^2 \neq 13$
- Meteor example:
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  $\langle q \rangle \neq \langle a \rangle \cdot (1 - \langle e \rangle)$

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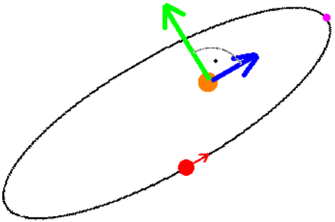
Conclusion:
mean orbit described above is set of parameters which does not define a real orbit.

Heliocentric vectorial elements

- Angular momentum ***h***
- Lenz vector ***e***

These vectors are perpendicular to each other. There are 5 independent components.



Method description

- Take at least 7 orbits
- Pre-integrate orbits into one epoch
- Calculate heliocentric vectorial elements ($\boldsymbol{h}$, $\boldsymbol{e}$, E) for each orbit
- Average vectorial elements constraining $\boldsymbol{h}$ and $\boldsymbol{e}$ are perpendicular to each other and equation for energy is satisfied
- Calculate heliocentric orbital elements (q, e, i, ω, Ω)
- Difference between obtained values can be as big as 2 AU and 0.5 deg in angular elements

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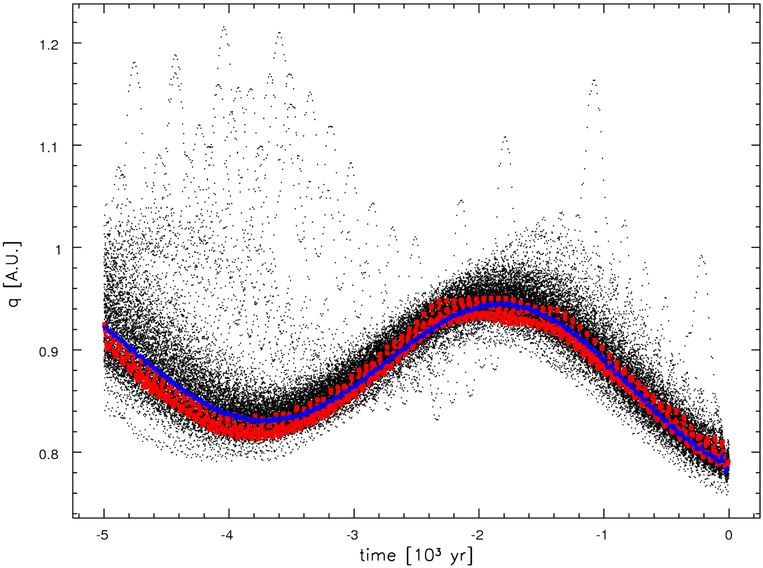
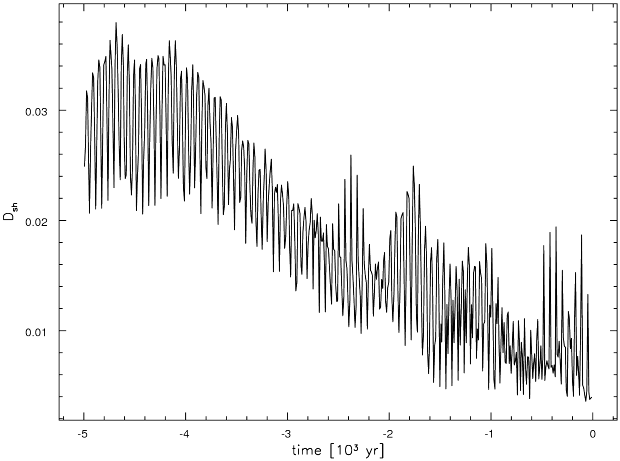
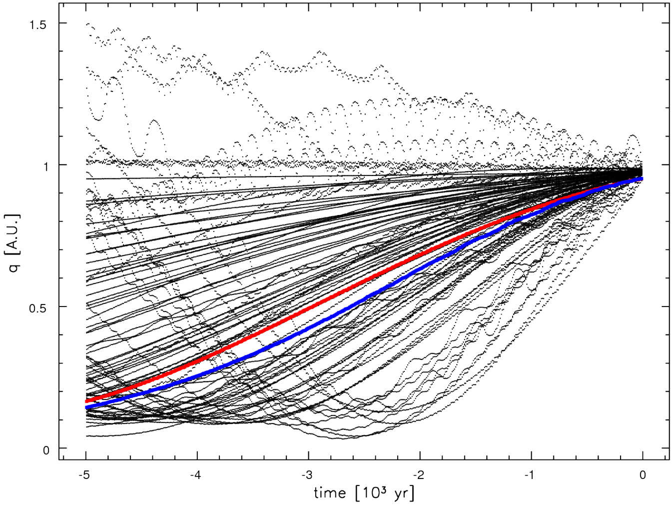
- 500 clones of Neuschwanstein meteorite consistent with uncertainties of observed orbital parameters.
- Each meteorite was integrated for 5000 years.
- For each 10 year interval orbits were averaged using Jopek *et al.* method (blue line).
- Also best-fitting orbit was integrated (red line).


q [A.U.]
1.2
1.1
1
0.9
0.8
-5
-4
-3
-2
-1
0
time [10^3 yr]

D-criterion for integrated mean orbit and mean of integrated orbits



86 Quadrantids



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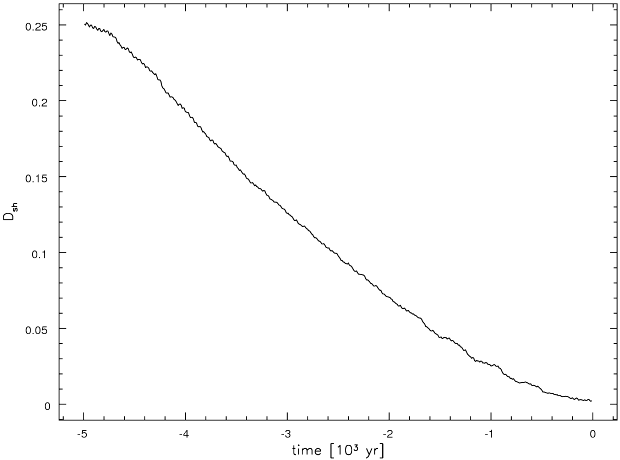

Conclusions

- One should use proper method of mean orbit calculation.
- For integrated clones of Neuschwanstein mean orbit is close to most probable orbit.
- Quadarantids: calculation of mean orbit after integration of orbits gives spurious results.

References:
Jopek, T. J., Rudawska, R. & Pretka-Ziomek, H. 2006, *MNRAS* **371**, 1367